Dynamická únosnost a životnost

Lekce 1
-
Elementary Fatigue Analysis

Jan Papuga

Jan Papuga

- living in Kunovice, Czech Republic
- PhD in Applied Mechanics 2006
- working at
  - Czech Technical University in Prague
  - Evektor spol. s r.o. (aircraft producer)
- previous jobs – Kappa 77 (producer of ultralight airplanes), SKODA VYZKUM (VZU Plzen now), ITER facility in Cadarache (France)
- developer of PragTic fatigue solver – www.pragtic.com
- initiator and leader of FADOFF consortium – www.fadoff.cz
- chairman of "Workshop on Computational Fatigue Analysis" (3-day educative workshop, 6 volumes already)
- Manager
  - "Damage Tolerance – Methods and Applications 2011" – a 11-day course in spring 2011
  - 6-day course „Design and Damage Tolerance for Aeronautical Engineers" in 2014
- co-chairman of Variable Amplitude Loading Conference (VAL2015) – Prague, March 2015

Recommended sources

- http://www.kmp.tul.cz/content/interaktivni-studijni-materialy
  - lectures
- Reading
  - Jágrová, J., Čapek, L.: Dynamická únosnost a životnost, skripta TUL FS, 2013
  - Růžička, M-Hanke.M.-RostM.: Dynamická pevnost a životnost. skripta ČVUT v Praze, 2. vyd., 1992.
  - Růžička,M, FidranskýJ. : Pevnost a životnost letadel, skripta ČVUT v Praze, 2000 (mechanika.fsid.cvut.cz)
  - Haibach,E.: Betriebsfestigkeit. 3.Auflage., Springer, 2006.
  - Schijve, J.: FatigueofStructuresand Materials. Springer, 2009.

Limit state design

Strength limit states (Mezní stavy pevnosti)

- Static strength
- Plasticity
- Stability, buckling
- Brittle fracture
- Creep
- Fatigue
- Thermal shock
- …

Serviceability limit states (MS funkční způsobilosti)

- Elastic and plastic deformation
- Shock
- Dynamic response
- Wear
- Corrosion
- …

+ all combinations of aforementioned limit states

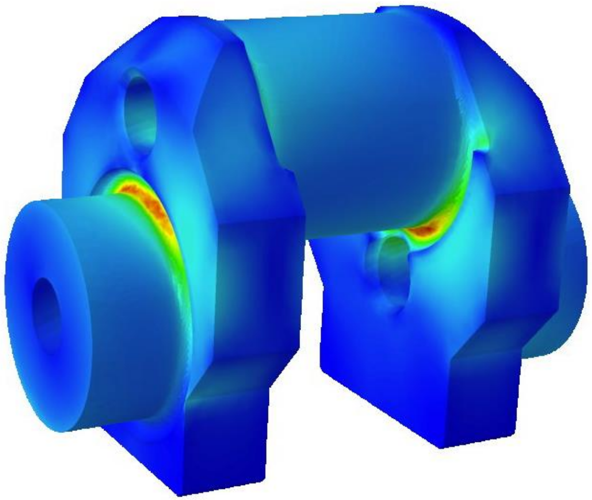
Theory of limit states – Application in

Design (i.e. a priori)

- Structure design
- Optimization
- Technology selection
- Setting service limitations

Service (i.e. a posteriori)

- In-service inspection
- Failure
- Accidents



www.reskof.cz

Fatigue process vs. Static load

90% of in-service failures are caused by fatigue damaging

Fatigue Damage

Caused by variable loading

- Usually. the damaging process cannot be seen by naked eye, but there are some exceptions:

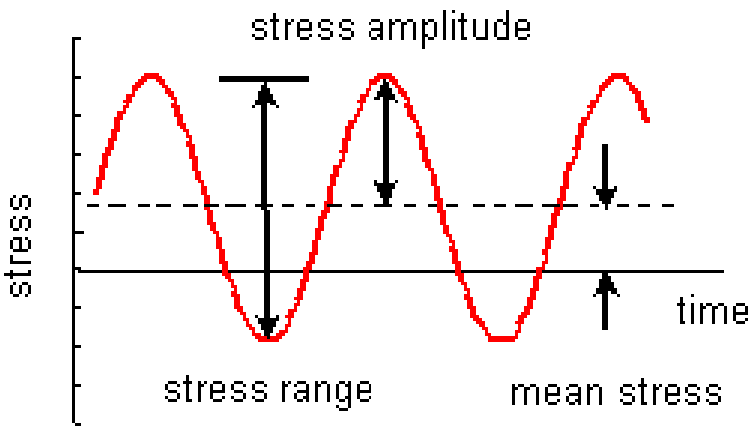
http://www.youtube.com/watch?v=3mclp9QmCGs

Cyclic process

Stress Amplitude. Sa $S_a = \dfrac{S_{\max imum} - S_{\min imum}}{2}$

Mean Stress. Sm $S_m = \dfrac{S_{maximum} + S_{minimum}}{2}$

Stress Range. ΔS $\Delta S = S_{maximum} - S_{minimum}$



Two types of fatigue loading regimes

stress amplitude loading control - *soft loading*
strain amplitude loading control - *hard loading*.

Wilhelm Albert (1829-1837)

- **1806 starts in a mining and forestry office in Clausthal**
- **From 1836, he manages all mines in Harz area**
- **1829 – observes. studies and describes damaging of chains serving for mining elevators caused by repeated loading of a small magnitude**

Probably the first written note on fatigue process

On “the first note”

- **Albert was the first to note something like that in a written text**
- **But:**

So long you go for water with a jug, that its handle breaks.

Czech proverb on fatigue process, my translation

- **In English:**

It is the fate of glass to break.

Well, that can be a static process, not fatigue

The jug goes to the well until it breaks.

Quite rare proverb, is it the same? It can be even a static process…

If starting with history…

http://courses.washington.edu/mengr541/ramulu/541/notes/notes1.pdf

The Basic Evolution Rule

1. You don't learn from making mistakes!
2. You learn from getting caught making mistakes!

Dan Lingenfelser. Caterpillar at SAE FD&E meeting. Fall 2007

Scale of the Mistake Matters

It could be that the purpose of your life
is only to serve as a warning to others

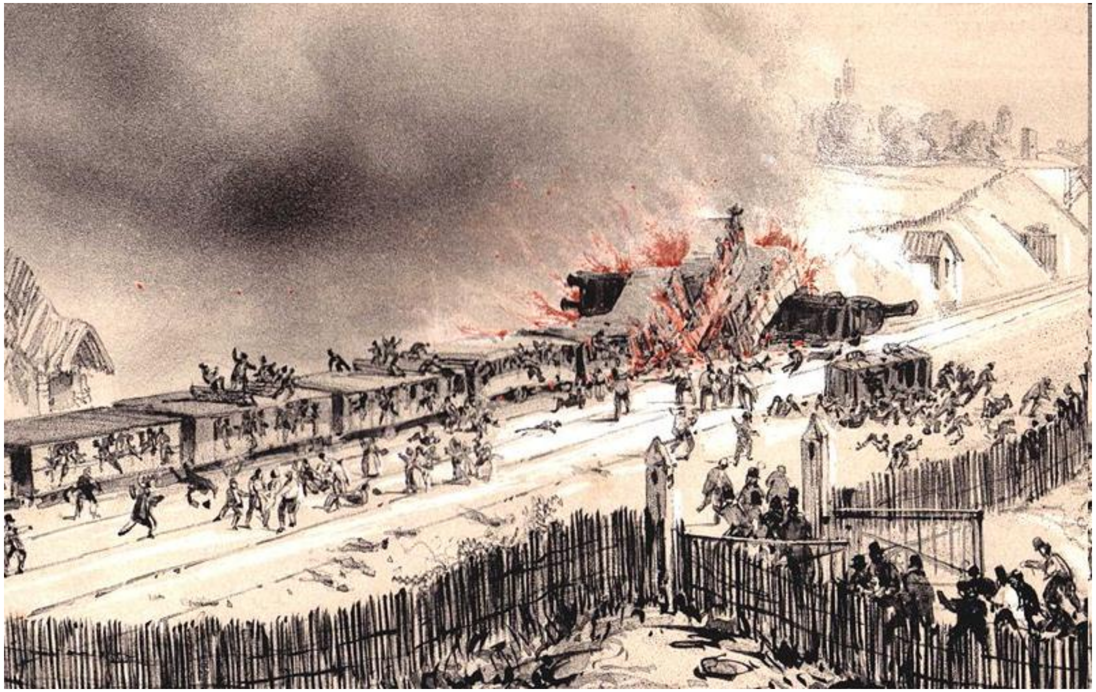
Dan Lingenfelser. Caterpillar at SAE FD&E meeting. Spring 2008

Train disaster in Versailles (1842)

Cause: Broken axle of the first locomotive

Result: Second machine and several wagons piled up over it and caught fire

- **Death of famous discoverer Dumont D'Urville and his family**

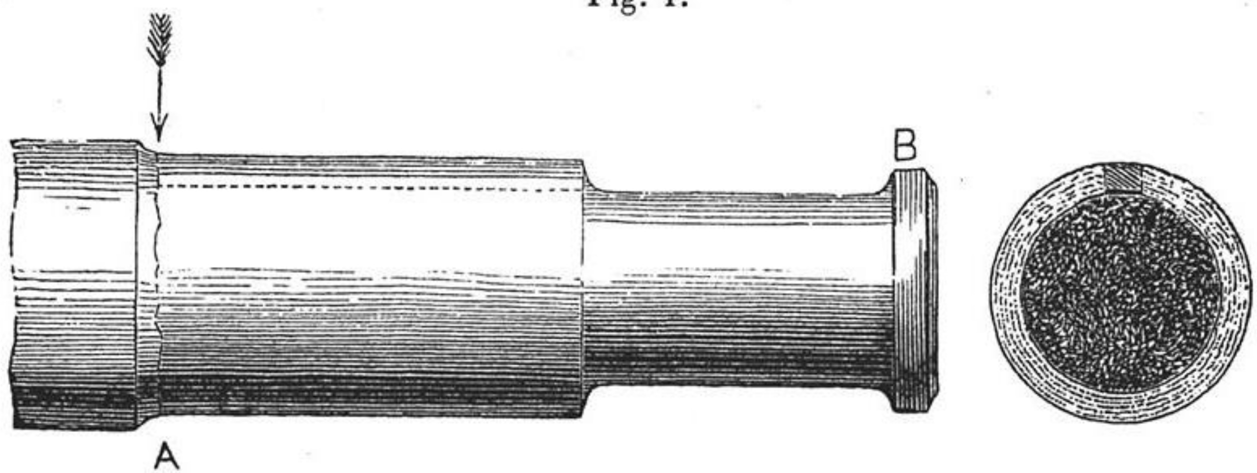


http://catskillarchive.com/rrextra/wkbkch06.Html

William J. M. Rankine (1840’s)

- **Famous Scottish physist and mechanist (Rankine cycle)**
- **Studies broken axles of locomotives and accents the effect of stress concentration**
- **Mostly ignored – the mainstream opinion is that the repeated loading causes recrystallization and strengthening of the structure**

Fig. 1.

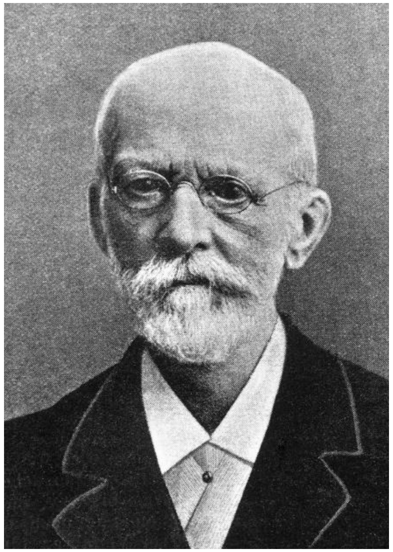


John Braithwaite (1854)

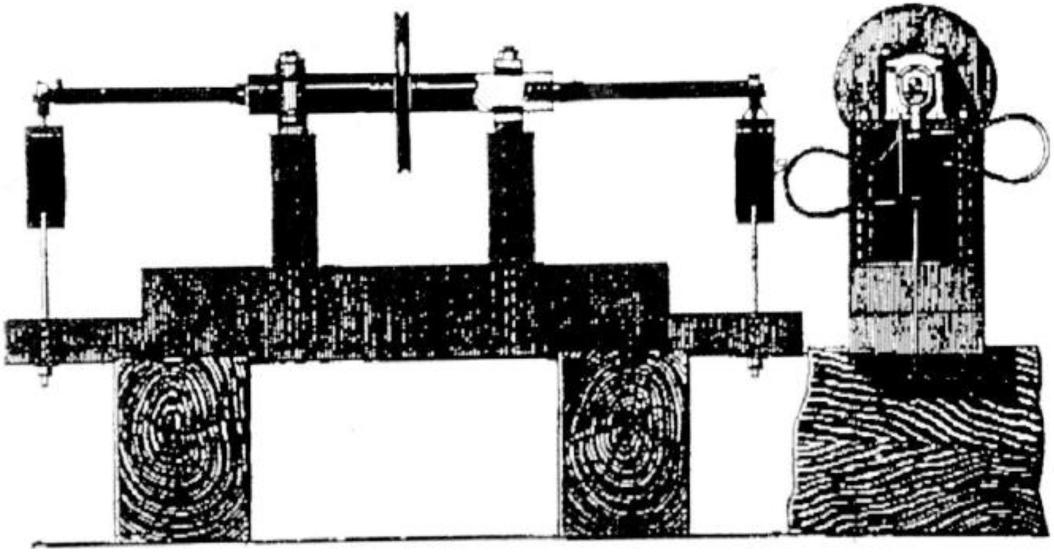
- **Engineering family with roots in the 17th century**
- **Together with Ericsson. he built the first locomotive able to reach 1 mile per one minute**
- **1854: The first time „fatigue“ term was used in relation to dynamic loading of mechanical structures:**

F. Braithwaite. (1854). "On the fatigue and consequent fracture of metals". *Institution of Civil Engineers. Minutes of Proceedings*. 463–474.

August Wöhler (1850-1870)



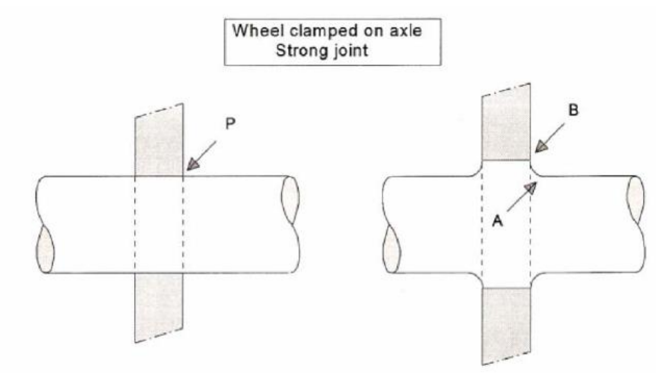
- **First systematic research, fatigue machine**
- **Introduces the term fatigue limit, describes the higher effect of the stress range in comparison to maximum stress**
- **Studies fatigue damaging of railway axles**



August Wöhler (1850-1870)

Studied fatigue failures on railway axles

- wheels shrink fitted on the axles

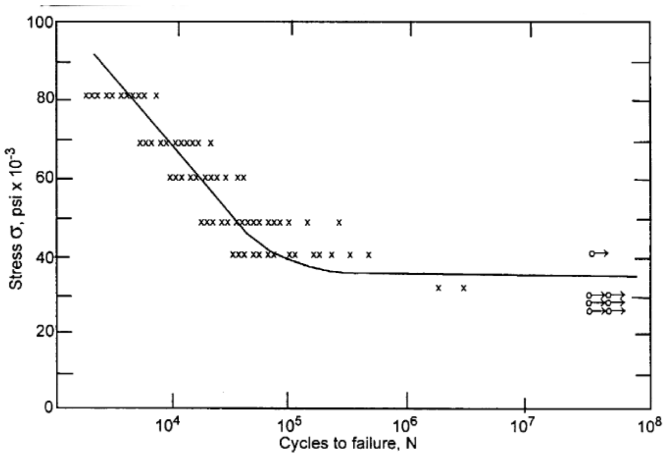


Reasons:

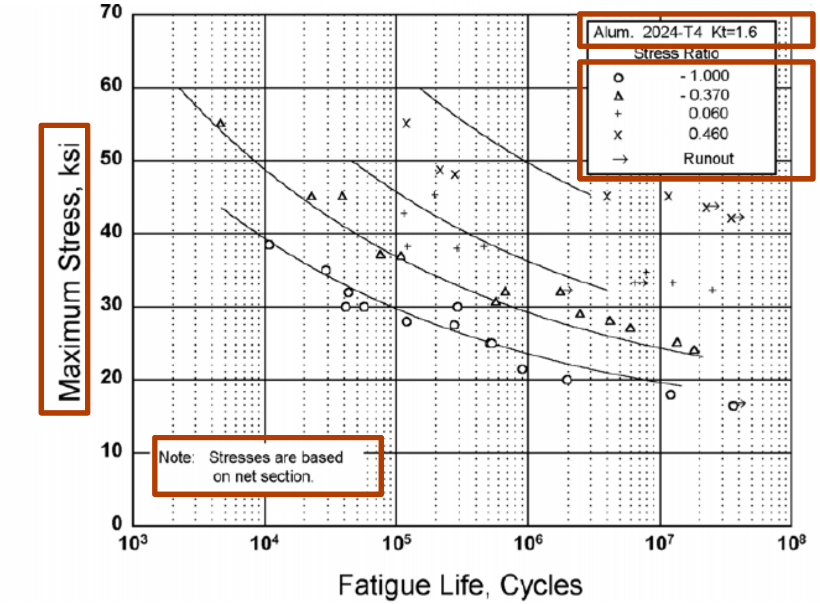
- High peak stresses at the joint
- Fretting fatigue in the joint induced by bending of the axles

Another outcome:

- Fatigue curve (Wöhler curve. S-N curve)



S-N curve



Rice. R.C.; Jackson. J. L.; Bakuckas. J.; Thompson. S.: Metallic Materials Properties Development and Standardization (MMPDS) [DOT/FAA/AR-MMPDS-01 Report]. FAA. Washington D.C. 2003.

What everything matters?

Product Form: Rolled bar, 1.125 inch diameter

Properties:	TUS, ksi	TYS, ksi	Temp., °F
	73	49	RT

Specimen Details: Semicircular
V-Groove, $K_t = 1.6$
0.450 inch gross diameter
0.400 inch net diameter
0.100 inch root radius, r
60° flank angle, ω

Surface Condition: As machined

Test Parameters:
Loading - Axial
Frequency - 1800 to 3600 cpm
Temperature - RT
Environment - Air

No. of Heats/Lots: Not specified

Equivalent Stress Equation:
$\text{Log } N_f = 12.25 - 5.16 \log (S_{eq} - 18.7)$
$S_{eq} = S_{max} (1-R)^{0.57}$
Std. Error of Estimate, Log (Life) = 0.414
Standard Deviation, Log (Life) = 0.989
$R^2 = 82\%$

Stress ratio

~ Coefficient of cycle asymmetry

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{\sigma_m - \sigma_a}{\sigma_m + \sigma_a}$$

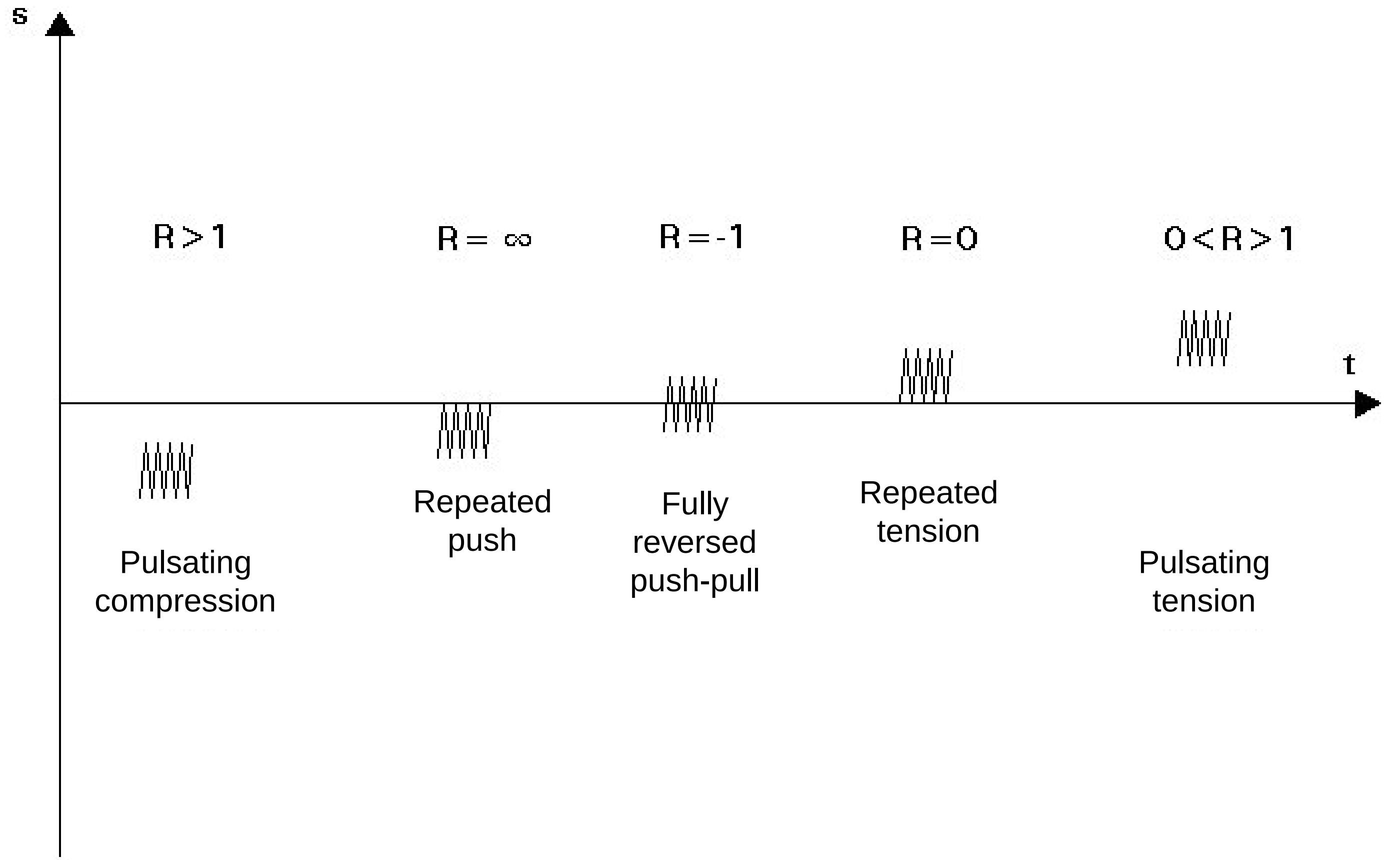


Figure 5 Stress cycles with different mean stresses and R-ratios.

S-N curve equation

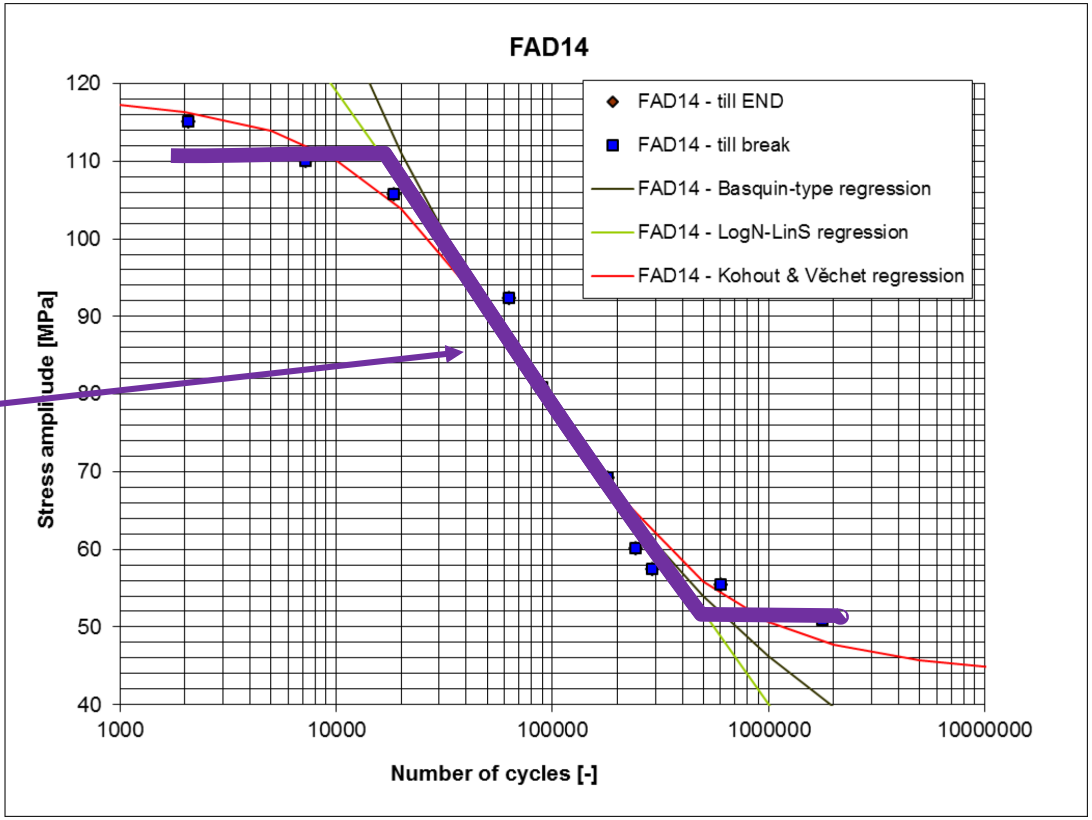
1910 – O. H. Basquin demonstrates on Wöhler's data qualities of his log-log relationship

$$\sigma_a = \sigma'_f \cdot (2N)^b$$

Another frequently used formula:

$$\sigma_a^w \cdot N = C$$

The only applicable area



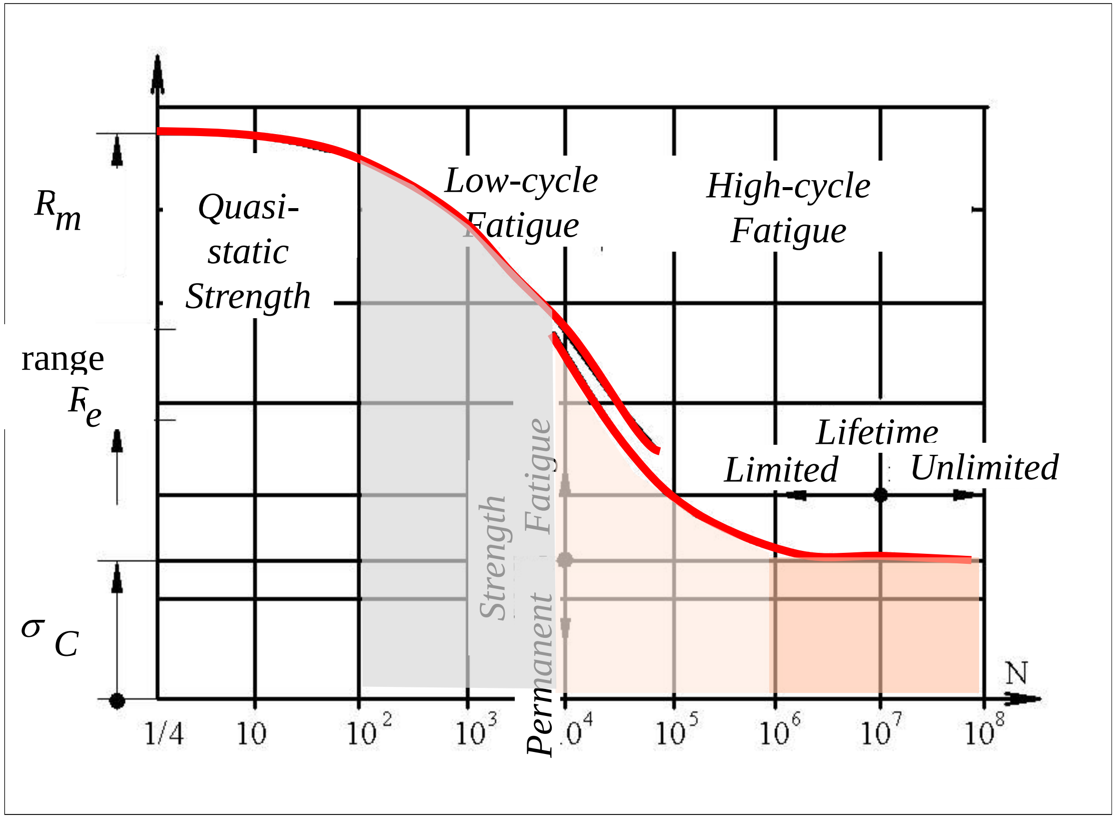
Fatigue damage domains

Quasi-static strength
(N<10^2 cycles)

Low-cycle fatigue
(10^2<N<$5\cdot10^5$ cycles)

High–cycle fatigue
($5\cdot10^5$< N<$2\cdot10^6$ cycles)

Very high-cycle (giga-cycle) fatigue (N>10^7 cycles)



Al and Mg alloys

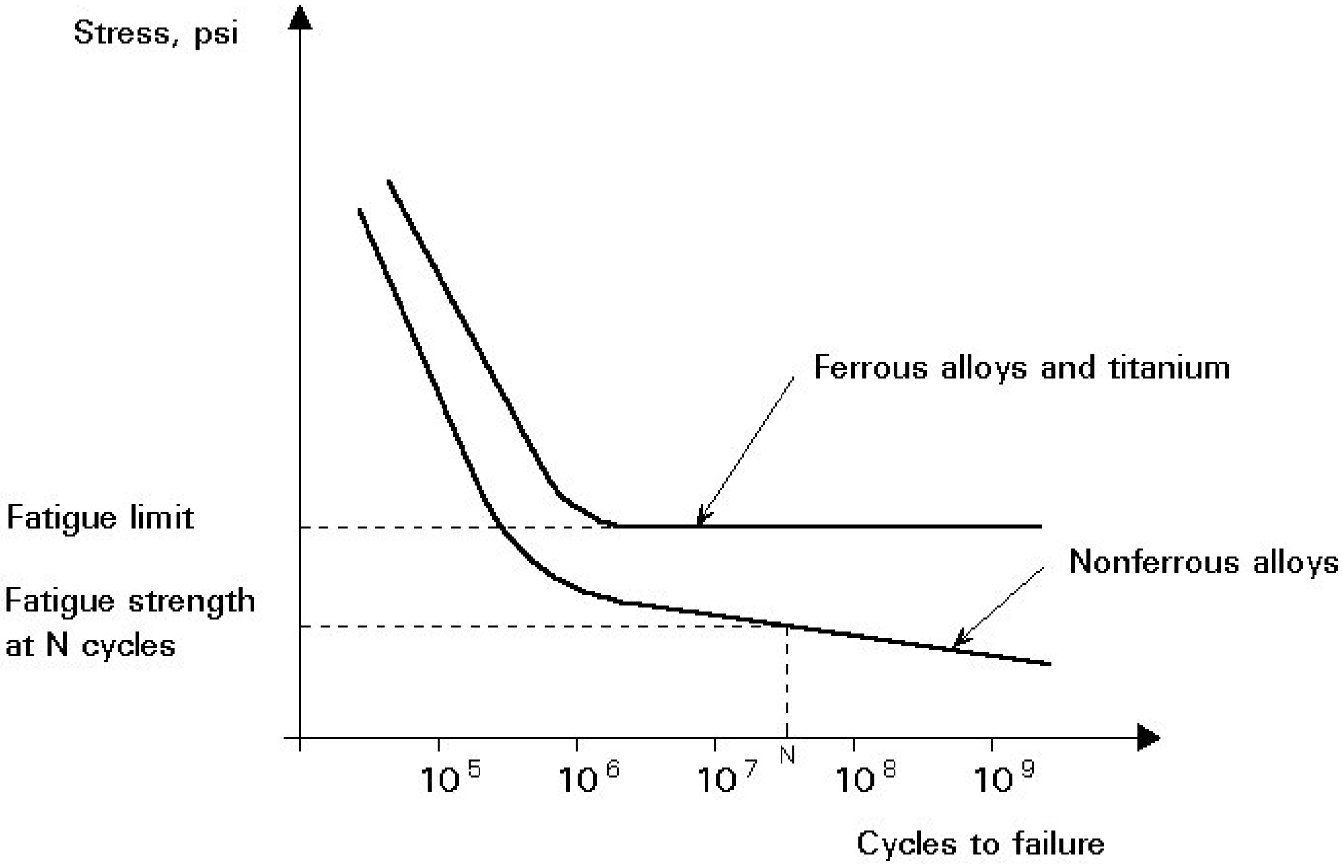
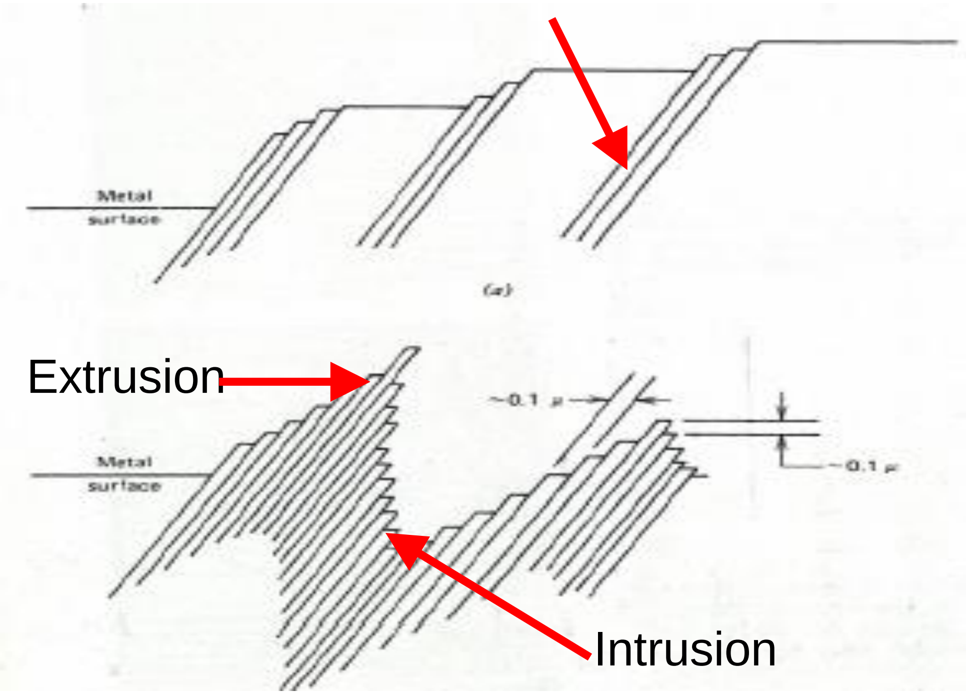
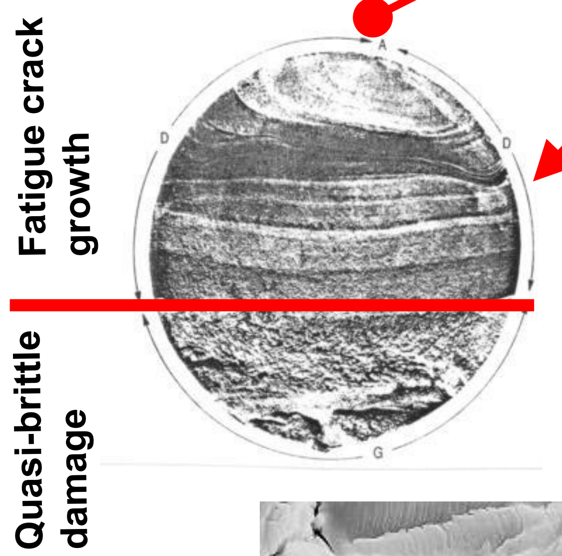


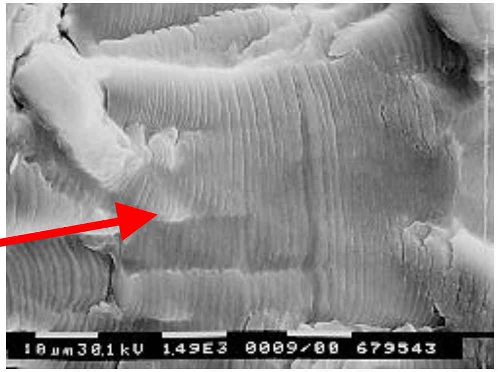
Figure 8 S-N curves with and without fatigue limits (schematic)

Initiation of cracks

1903 – Sir James Alfred Ewing shows that the origin of fatigue cracks can be traced to microscopic cracks

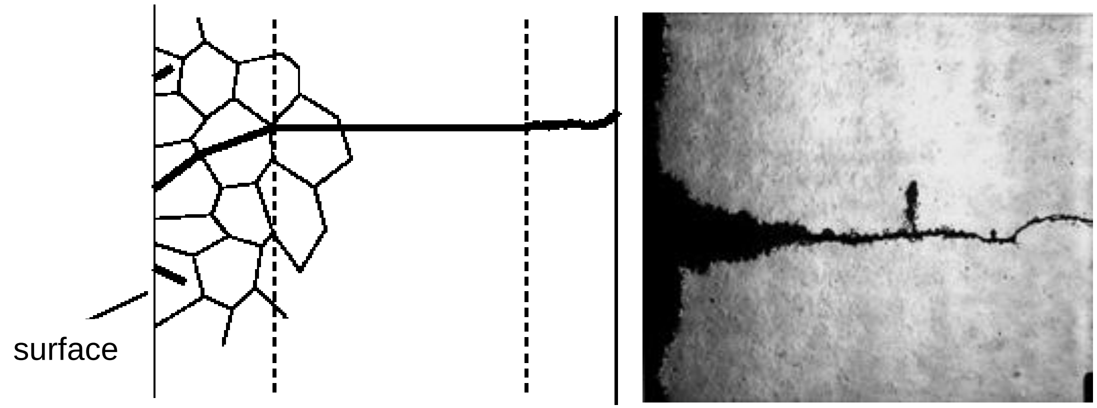


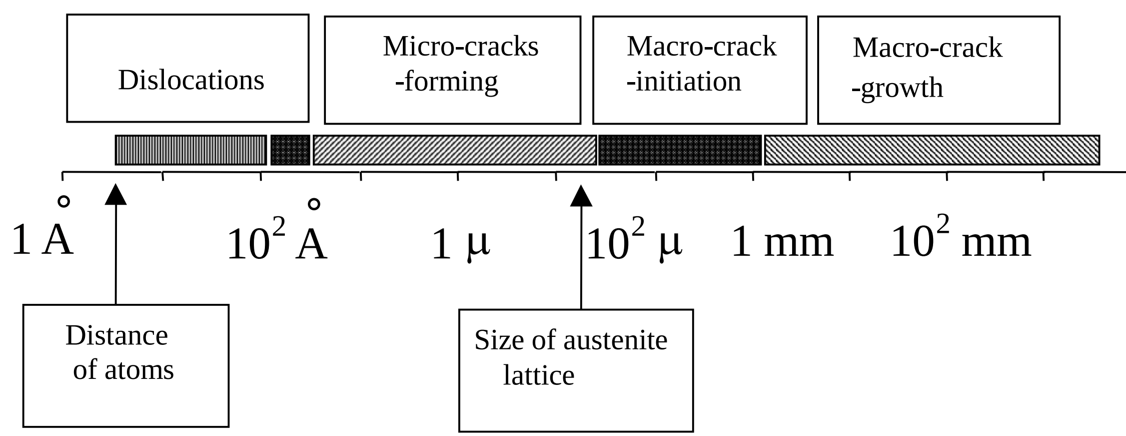




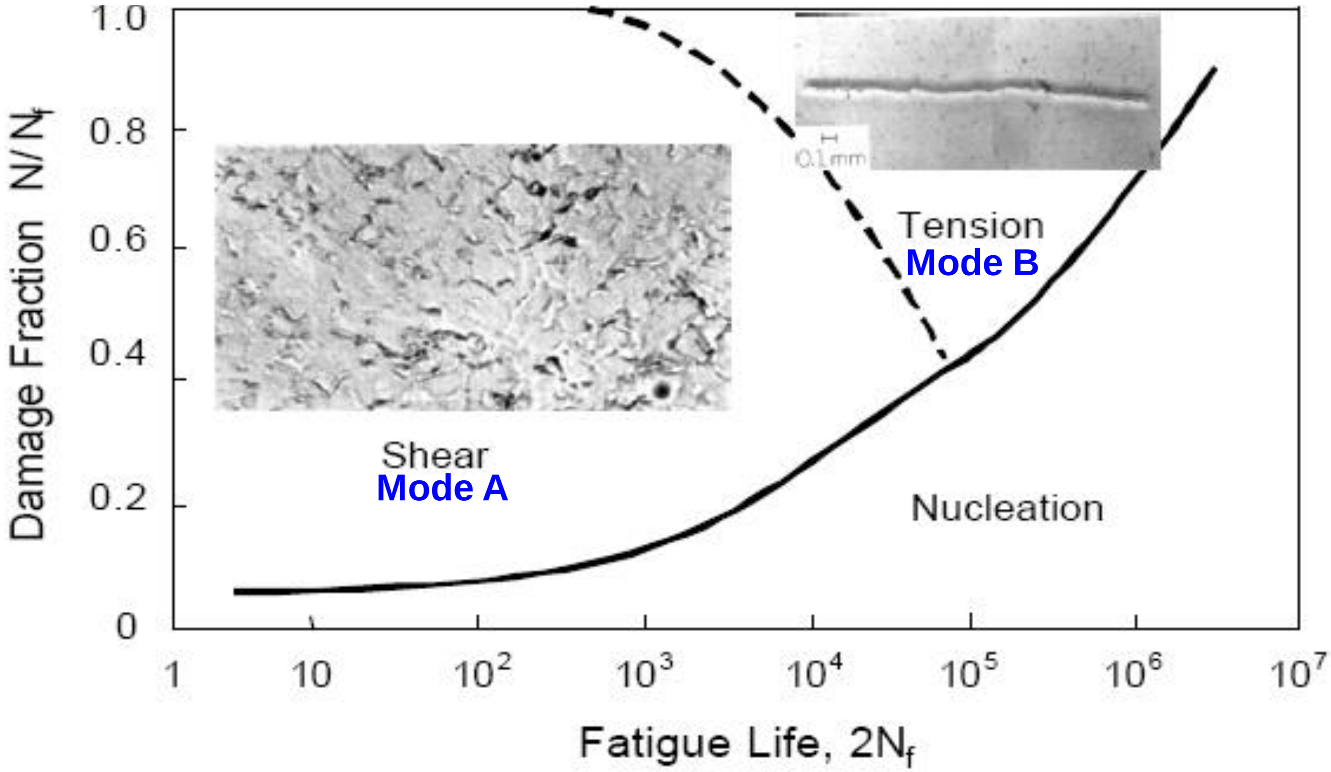
Fatigue process phases

- **Phase of cyclic behaviors changing** - change of metal structure in whole volume. Just few percent of specimen life.
- **Phase of fatigue crack nucleation** - local changes in surface layers of material caused by dislocation effect.
- **Phase of crack propagation -** stage of micro-crack growing in major crack and further crack growth.
- **Phase of final fracture -** high-speed quasi-brittle crack of residual section (fracture toughness exceeded or ductile crack at yield and strength limit exceeded)





Damaging mechanisms



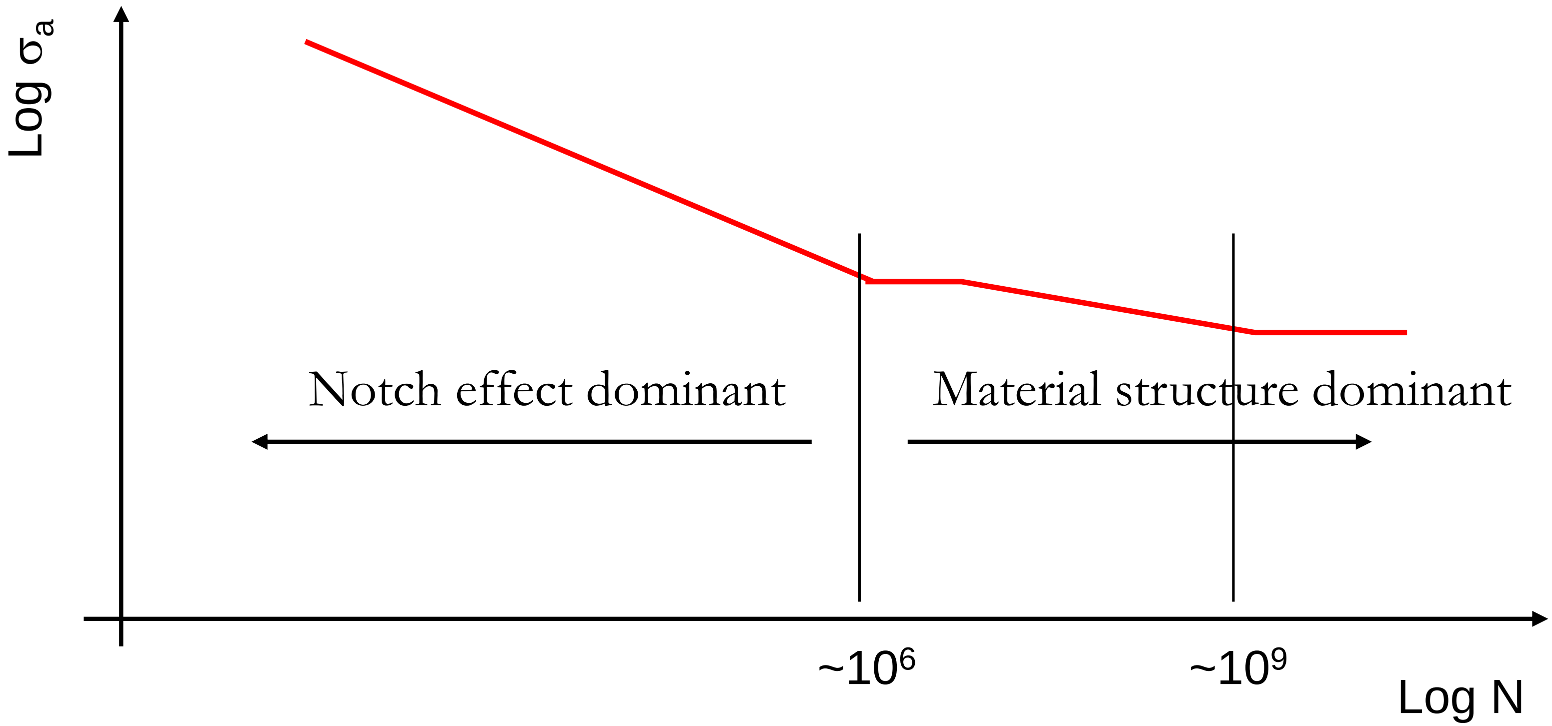
SOCIE. D. F.: Critical plane approaches for multiaxial fatigue damage assessment. In: Advances in Multiaxial Fatigue. ASTM STP 1191. Red. D. L. Dowell a R. Ellis. Philadelphia. American Society for Testing and Materials 1993. pp. 7-36.

High-cycle fatigue (HCF)

- Small loads = long lifetimes
- Small or negligible areas of plasticity
- Most of the lifetime spent on crack initiation
- Use of S-N curves preferred
- Design to unlimited lifetime is an option

Fatigue limit

- **Limit stress. that can be applied on the specimen without breaking it after any time**
- **Can be only conventional value in some cases (aluminium alloys)**
- **Important when designing to infinite life**
- **Giga-cycle fatigue (GCF) / Very high cycle fatitue (VHCF):**



Fatigue Limit Estimates

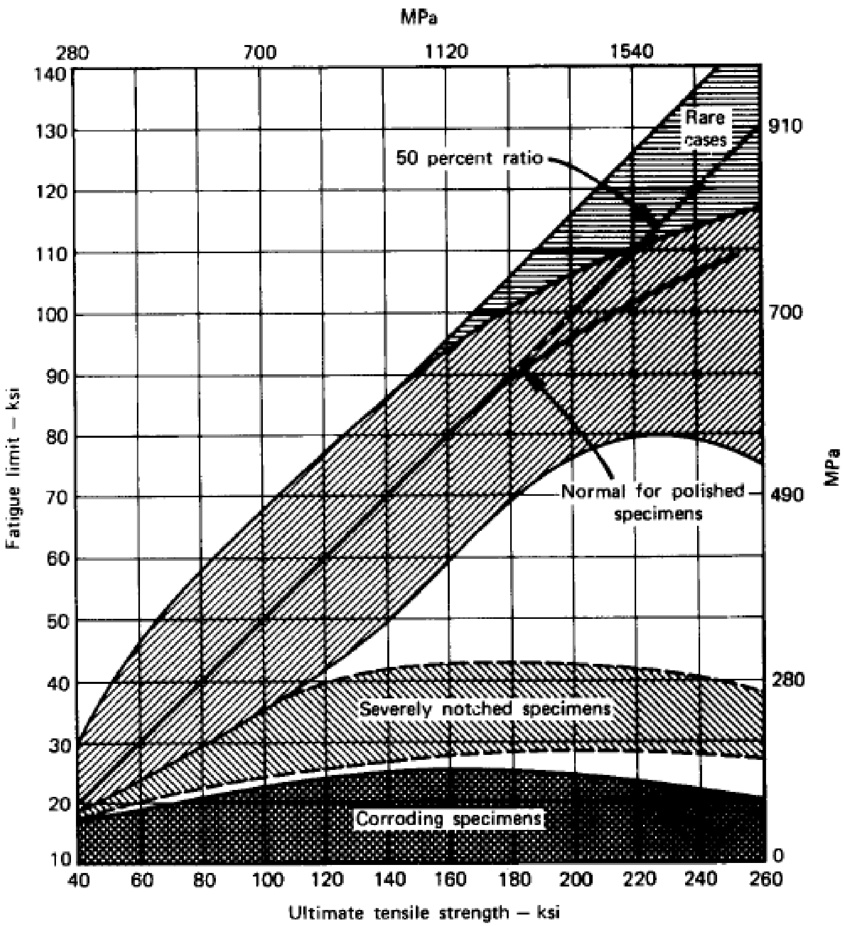
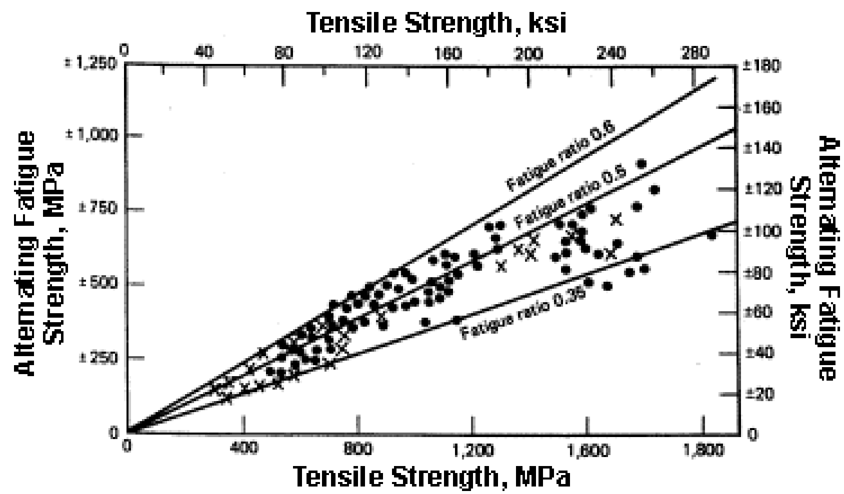


FIGURE 6-9

Relationship Between Fatigue Limit and Ultimate Strength for Steel Specimens (From *Steel and Its Heat Treatment*, by D. K. Bullens, John Wiley & Sons, New York, 1948, with permission of the publisher)



http://fatiguecalculator.com

STEELS:

S_f in tension ≅ 0.35 Rm

in bending = 0.43 Rm

in torsion≅ 0.25 Rm .

Fatigue Limit Estimates - FKM

FKM Guideline – German guideline defining recommended practice in static and fatigue analyses. both in nominal and local approaches

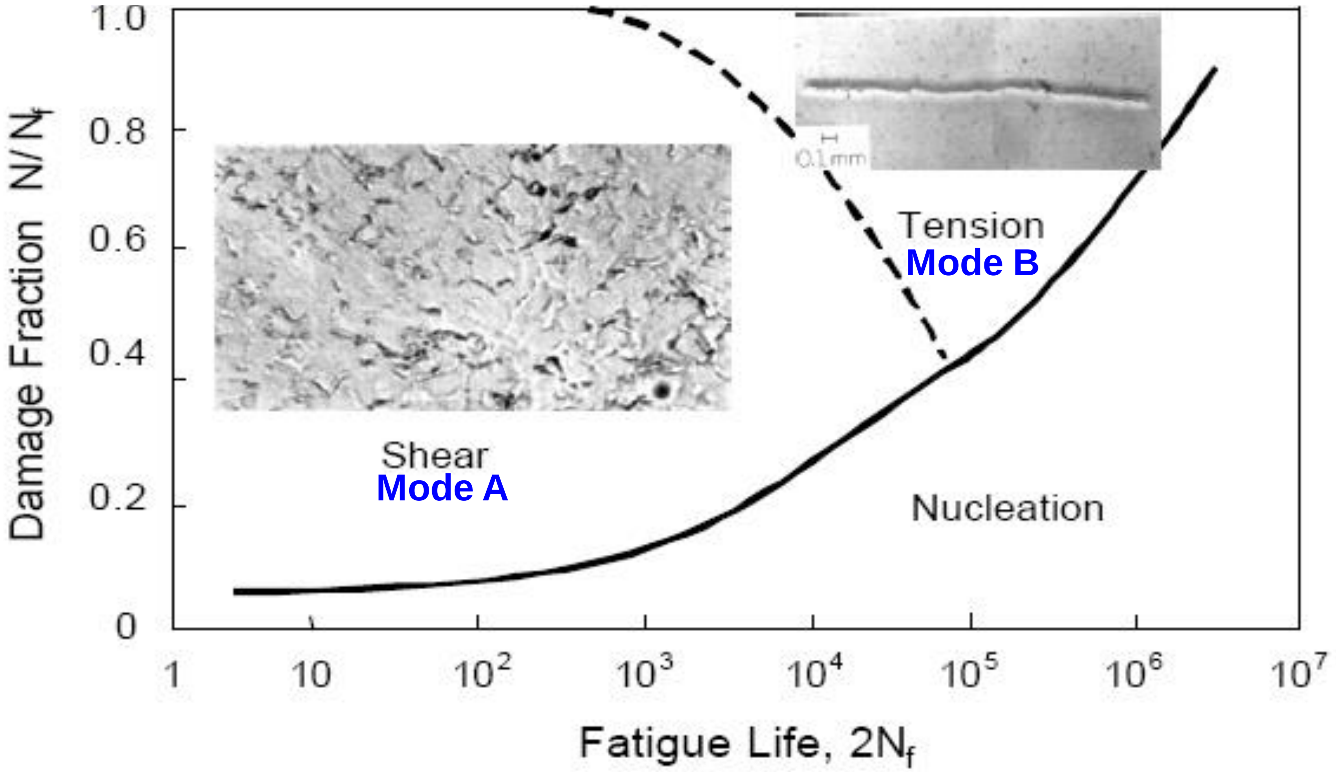
Material group	$f_{W.\sigma}$	$f_{W.\tau}$
Case hardening steel	0.4	0.577
Stainless steel	0.4	0.577
Forging steel	0.4	0.577
Other steels	0.45	0.577
Cast steel	0.34	0.577
Spheroidal cast iron	0.34	0.65
Ductile cast iron	0.3	0.75
Grey cast iron	0.34	1.0
Wrought aluminum alloy	0.3	0.577
Cast aluminum alloys	0.3	0.75

$$S_F = f_{W.\sigma} \cdot R_m$$

$$S_{F,t} = f_{W,\tau} \cdot S_F$$

FKM-Guideline: Analytical Strength Assessment of Components in Mechanical Engineering. 5th revised edition. Frankfurt/Main, Forschungskuratorium Maschinenbau (FKM) 2003.

Damaging mechanisms

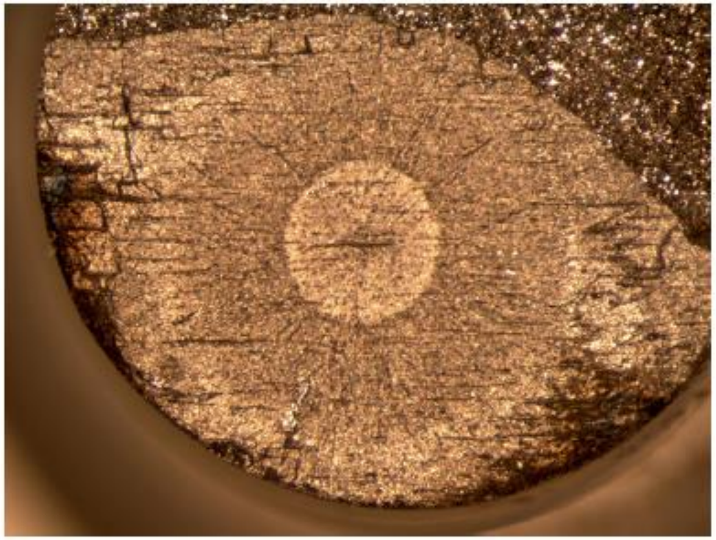


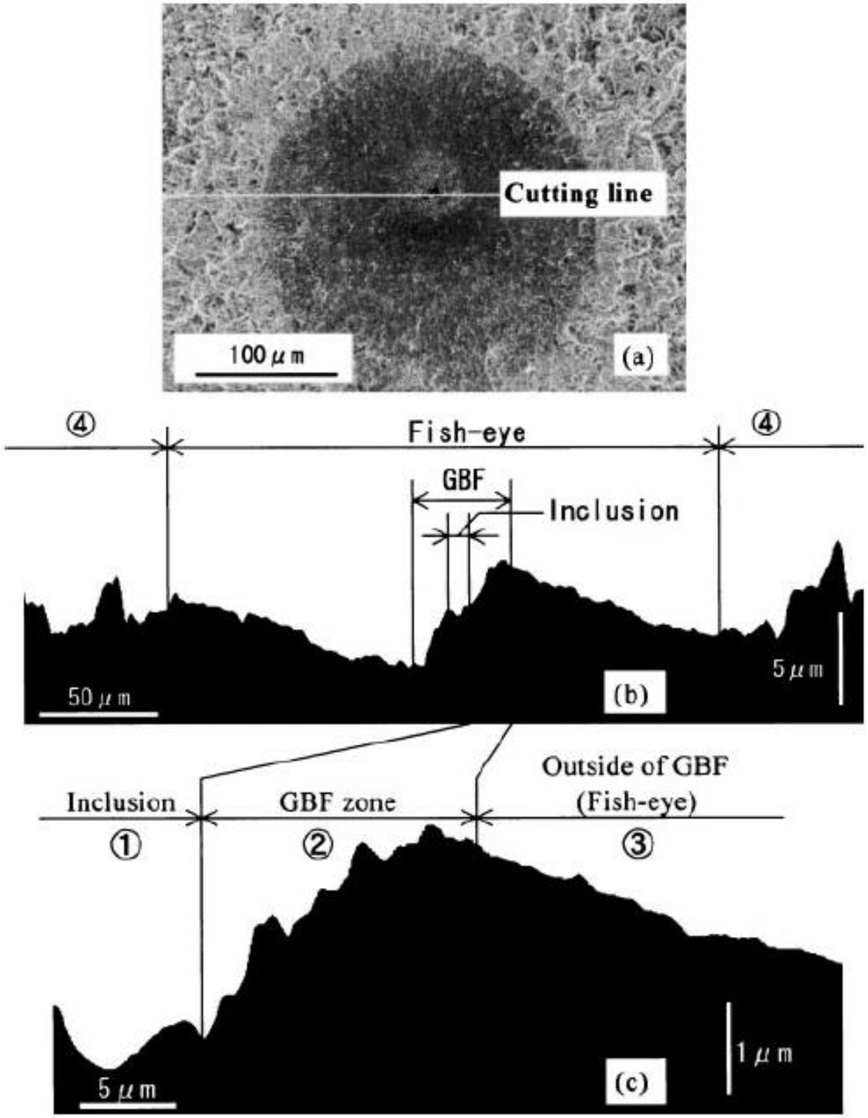
S-N curve shows the lifetime till the final rupture of the specimen (if not stated otherwise)

SOCIE. D. F.: Critical plane approaches for multiaxial fatigue damage assessment. In: Advances in Multiaxial Fatigue. ASTM STP 1191. Red. D. L. Dowell a R. Ellis. Philadelphia. American Society for Testing and Materials 1993. pp. 7-36.

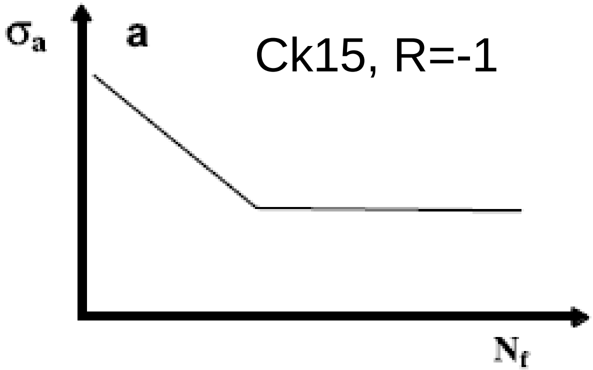
Gigacycle Fatigue

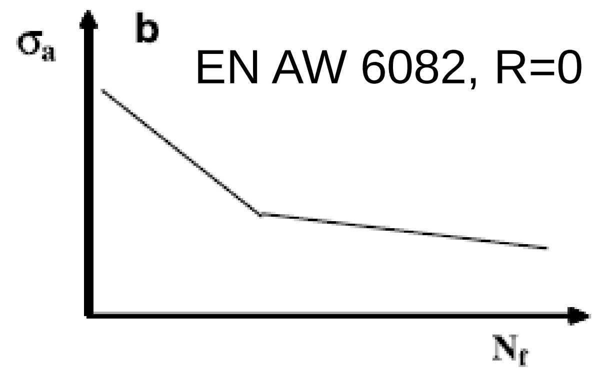
- Cracks can start from the inner of the cross-section
- Related to material non-homogeneity



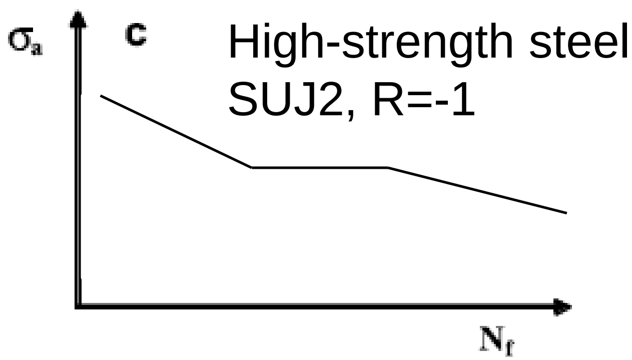


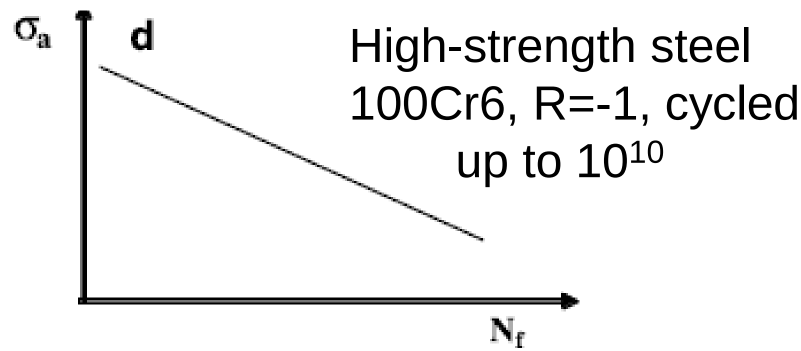
Gigacycle Fatigue





Pyttel, B.; Schwerdt, D.; Berger, C.: Very high cycle fatigue – Is there a fatigue limit? Int J Fatigue 33 (2011), pp. 49-58.





Design code	Material	Assumption
FKM-Guideline [17]	Steel, cast iron	$k = 5$ for LCF, fatigue limit from $N_k=10^6$,
	aluminium alloys	$N_k=10^6$, $k^*=15$, fatigue limit from $N_k=10^8$
IIW-Recommendation [19]	Steel, aluminium alloys, welded	$N_k=10^7$, $k^*=22$, no fatigue limit
Eurocode 3 [18]	Steel	Fatigue limit from $N_k=5\text{x}10^6$
Eurocode 9 [20]	Aluminium	Fatigue limit from $N_k=10^8$

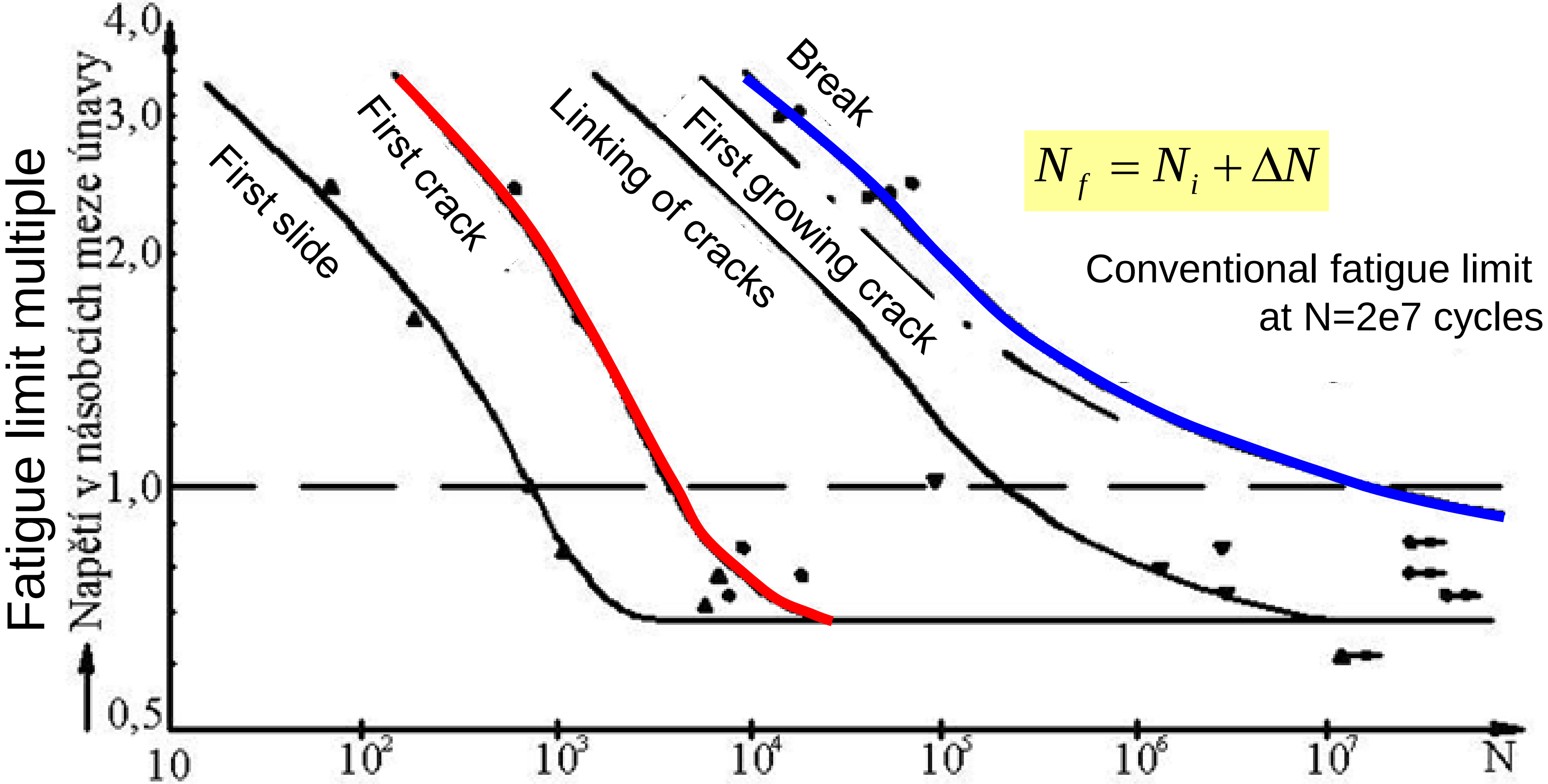
Gigacycle Fatigue

Sonsino CM. Course of SN-curves especially in the high-cycle fatigue regime with regard to component design and safety. Int J Fatigue 2007; 29:2246–58.

Material	N_k	k^*	Decrease per decade in %	$1/T_\sigma$
Steel, not welded	5x105 - high-strength 2x106 - structural steels	45	5	1.20
Steel, welded	$1\text{x}10^6$ - thermal stress relieved $1\text{x}10^7$ - high tensile residual stresses	45 22	5 10	1.50
Cast steel	$5\text{x}10^5$ - high-strength $2\text{x}10^6$ - medium-strength	45	5	1.40
Sintered steel	$5\text{x}10^5$ - high-strength $2\text{x}10^6$ - medium-strength	45	5	1.25
Cast nodular iron	$5\text{x}10^5$ - high-strength $2\text{x}10^6$ - medium-strength	45	5	1.40
Wrought Al alloys, not welded	$1\text{x}10^6$–$5\text{x}10^6$	22	10	1.25
Wrought Al alloys, welded	$1\text{x}10^6$ - low tensile resid. stresses $1\text{x}10^7$ - high tensile resid. stresses	22	10	1.45
Cast aluminium	$1\text{x}10^6$–$5\text{x}10^6$	22	10	1.40
Sintered aluminium	$1\text{x}10^6$	22	10	1.25
Wrought Mg alloys, not welded	$5\text{x}10^4$–$1\text{x}10^5$	45	5	1.20
Cast magnesium	$1\text{x}10^5$–$5\text{x}10^5$	45	5	1.30
Wrought Mg alloys, welded	$5\text{x}10^5$ - low tensile resid. stresses $1\text{x}10^7$ - high tensile resid. stresses	22	10	1.50

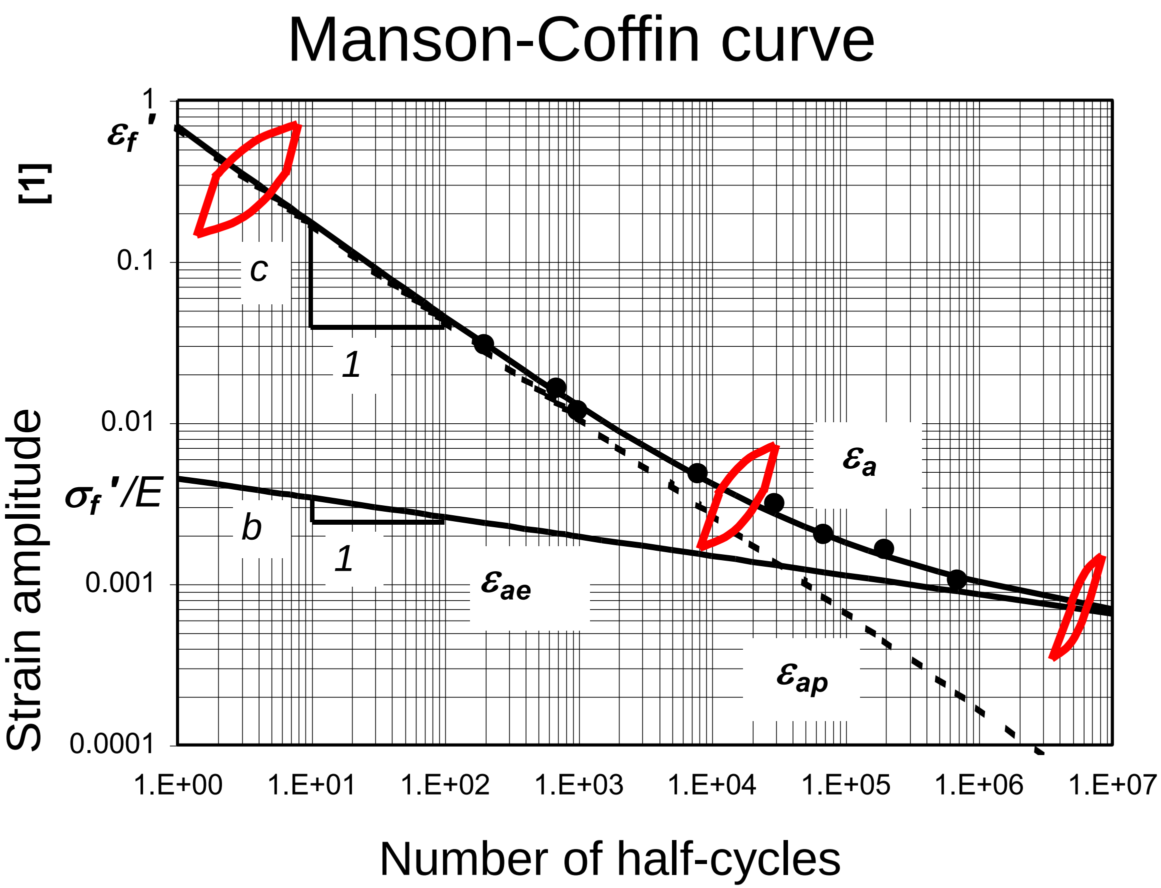
Explanation: N_k ~ break point; k^* ~ S-N curve slope after N_k; $T_\sigma = \sigma_a(P_s{=}90\%) / \sigma_a(P_s{=}10\%)$

Damage growth



Strain-based fatigue curve

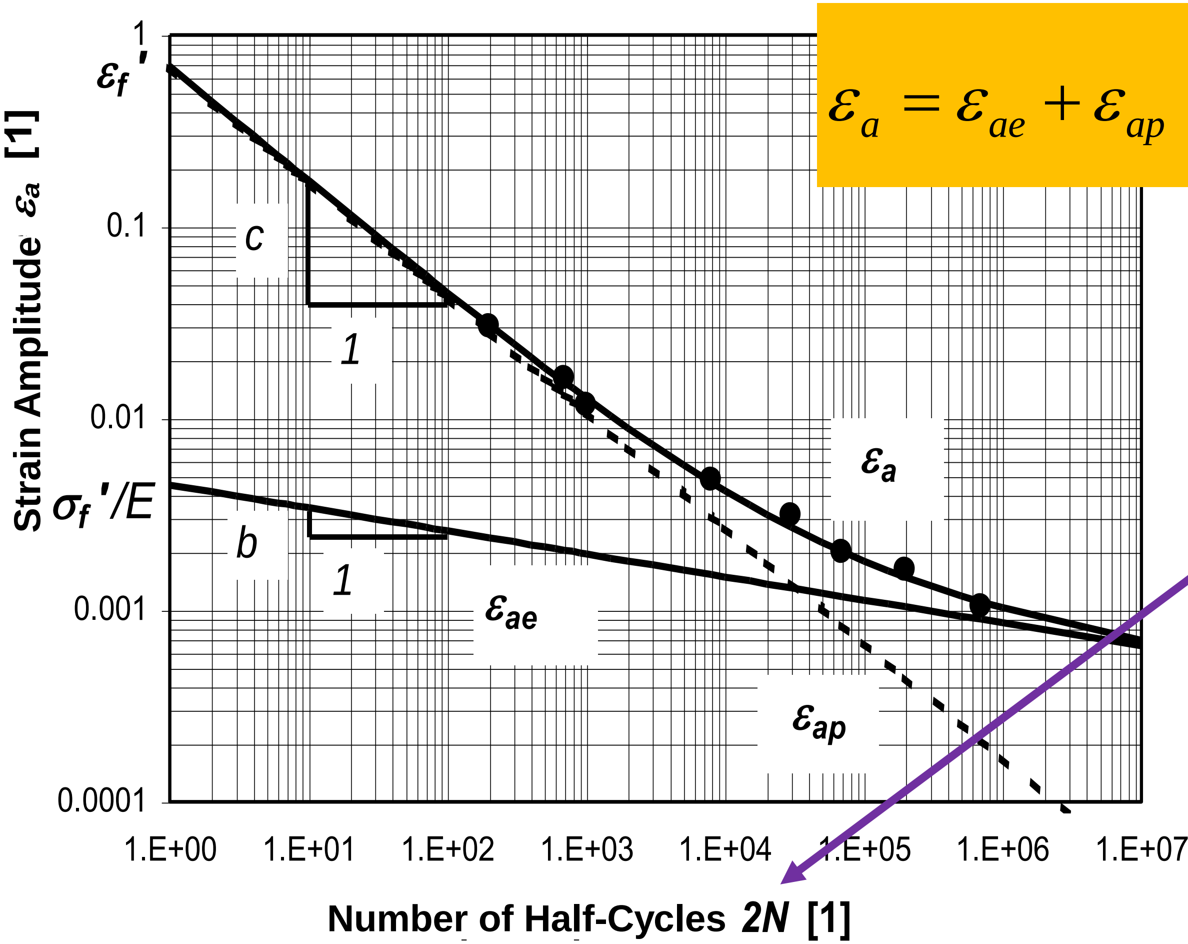
1954 – Wholly independently. L. F. Coffin and S. S. Manson show practical use of plastic strains in the crack tip for the fatigue crack growth



The curve relates to a moment of initiating a technical macro-crack

Manson-Coffin curve (e-N curve)

$$\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma'_f}{E}(2N)^b + \varepsilon'_f (2N)^c$$

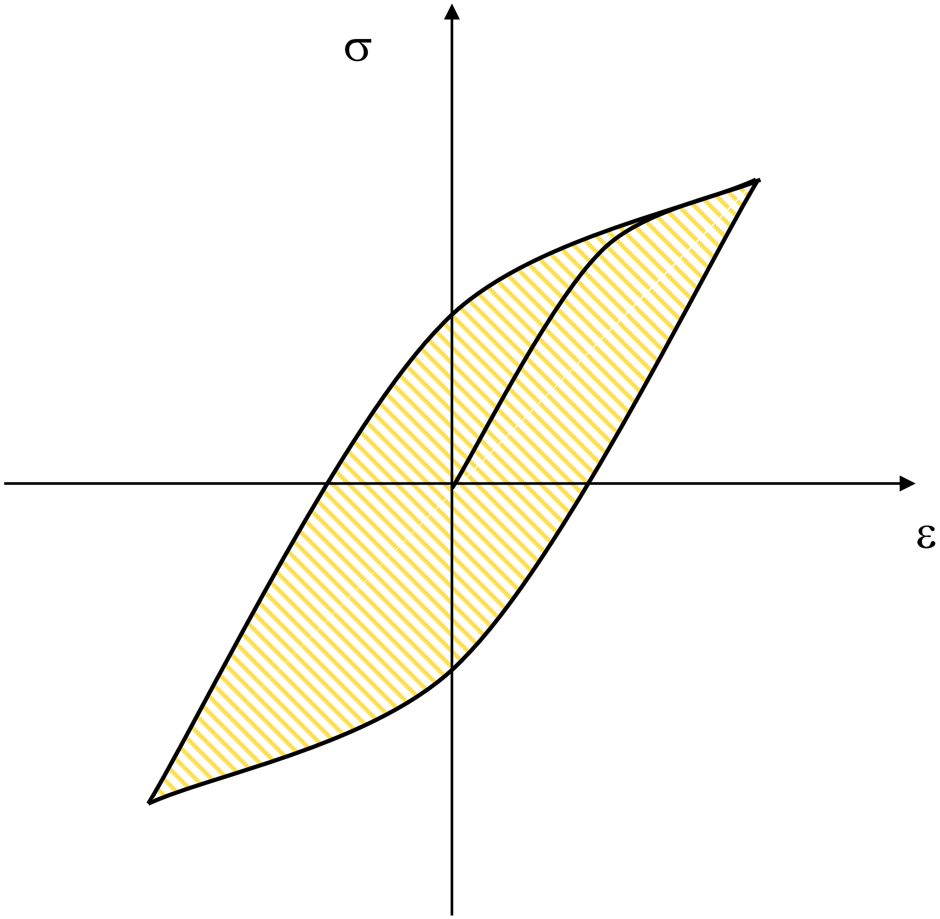


Take care!

σ'_f	fatigue strength coefficient
b	fatigue strength exponent
ε'_f	fatigue ductility coefficient
c	fatigue ductility exponent

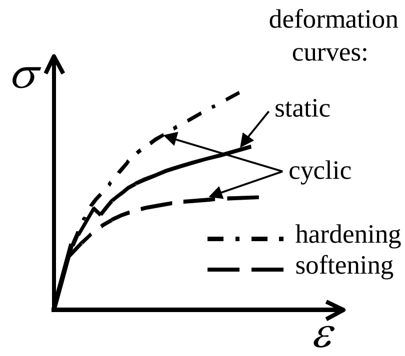
The criminal and its traces

Input strain energy => hysteresis loop

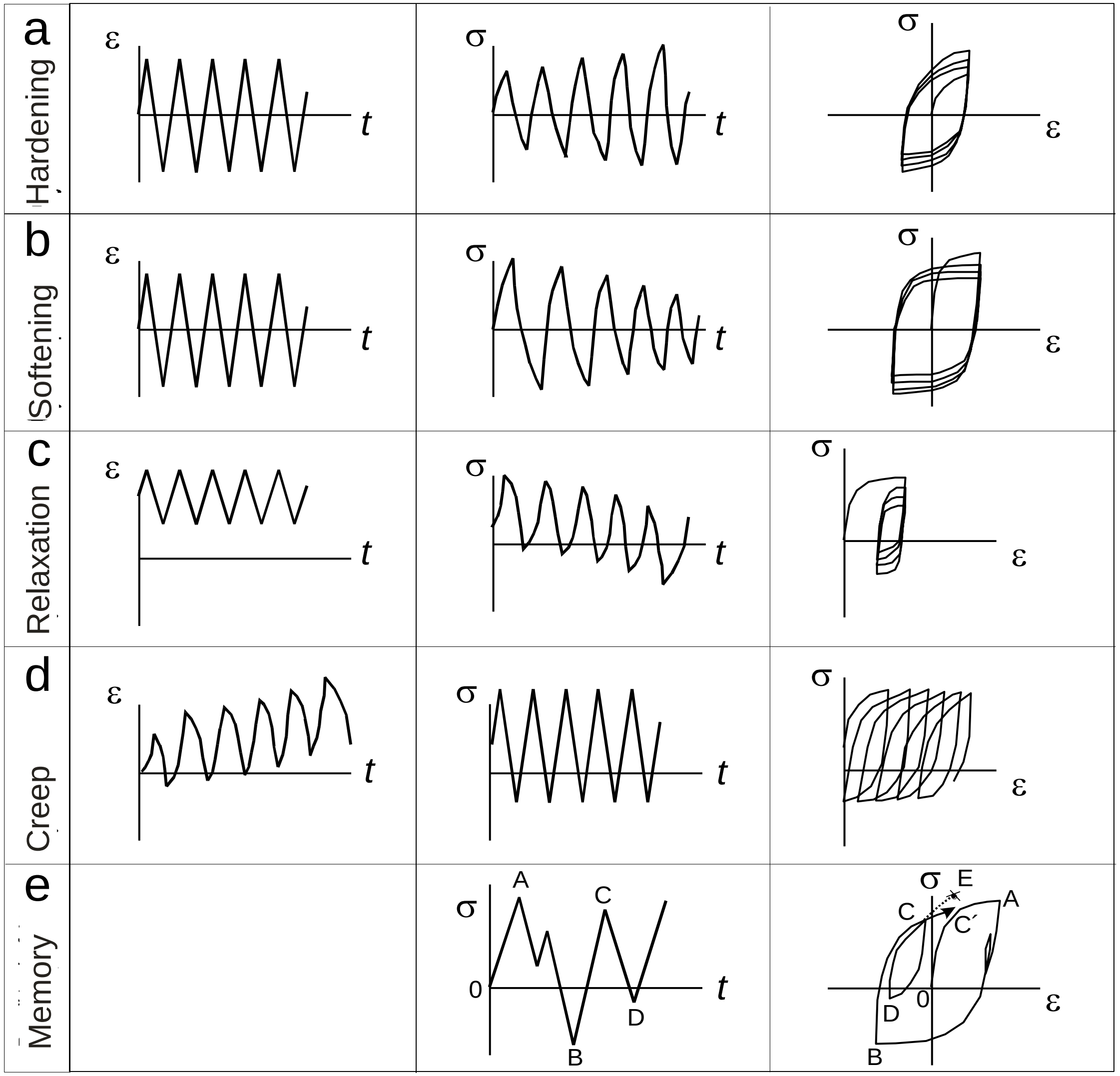


Even macroscopically elastic loading changes to elastic-plastic locally

Cyclic Material Behavior and Its Changes

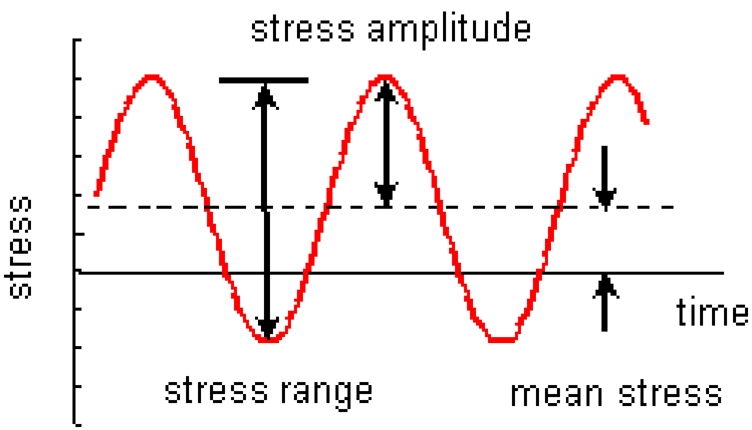


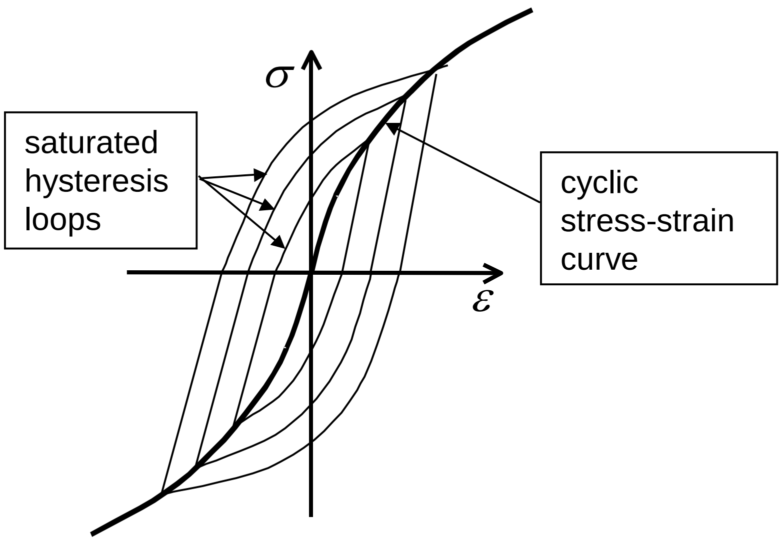
The material response needn't stabilize during whole life of the specimen



Cyclic stress-strain curve

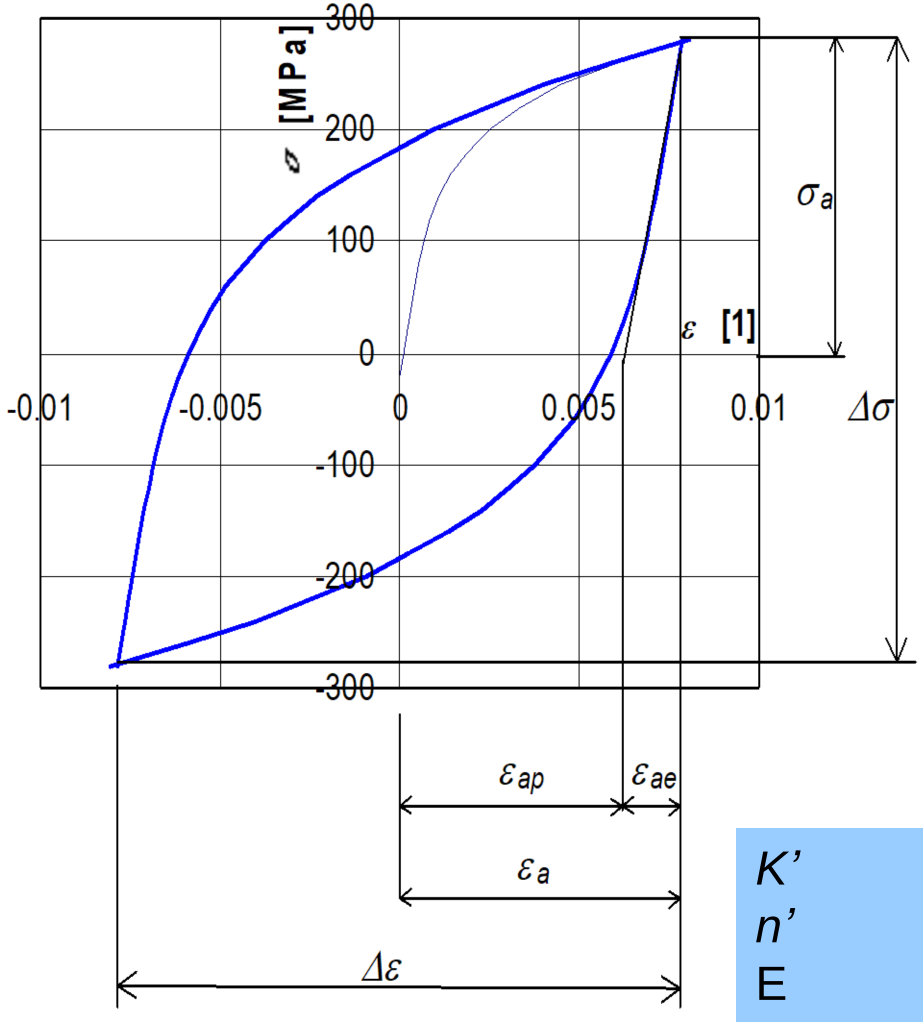
Input strain energy => hysteresis loop





The materials deformation during a fatigue test is measured in the form of a hysteresis loop. After some initial transient behavior the material stabilizes and the same hysteresis loop is obtained for every loading cycle. Each strain range tested will have a corresponding stress range that is measured. The cyclic stress strain curve is a plot of all of this data

Cyclic stress-strain curve formula



Static: Ramberg-Osgood formula

$$\sigma = K \cdot \varepsilon_p^n$$

Aproximation of the cyclic stress-strain curve

$$\sigma_a = K' \cdot \varepsilon_{ap}^{n'}$$

$$\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}}$$

K'	- cyclic strain hardening coefficient
n'	- cyclic strain hardening exponent
E	- Young's modulus of elasticity (tension)

M-C vs. R-O

$$\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}}$$

$$\sigma_a = \sigma'_f \cdot (2N)^b$$

$$\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma'_f}{E}(2N)^b + \varepsilon'_f (2N)^c$$

K' cyclic strain hardening koeff.
n' cyclic strain hardening exponent

σ'_f fatigue strength coefficient
b fatigue strength exponent

ε'_f fatigue ductility coefficient
c fatigue ductility exponent

6 material parameters. 4 independent: $K', \quad n', \quad \sigma'_f, \quad b, \quad \varepsilon'_f, \quad c$

$$\sigma_a = \sigma'_f \cdot (2N)^b = K'(\varepsilon'_f)^{n'} \cdot (2N)^{c \cdot n'}$$

$$n' = \frac{b}{c}$$

$$K' = \frac{\sigma'_f}{(\varepsilon'_f)^{n'}}$$

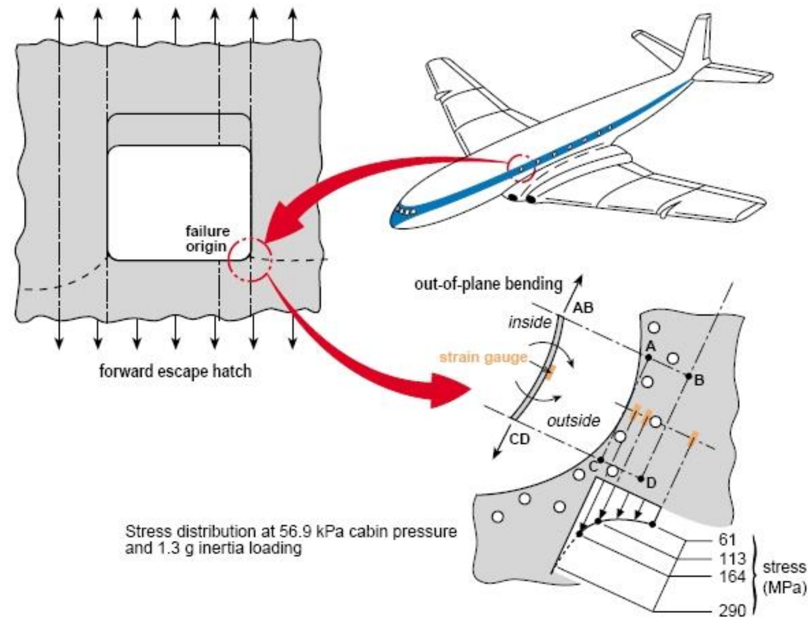
Low-cycle fatigue (LCF)

- **High loads = shorter lifetimes**
- **Larger areas of pronounced plasticity**
- **Relates only to the life till the crack initiation**
- **Use of M-C curves preferred**
- **Plasticity is an issue**

De Havilland Comet (1954)

First airliner able to fly in high altitudes

Two airplanes lost with 2 months (1286 a 903 take offs. nobody survived)



Reason

- Combined bending caused that the stress at the inner surface was higher than on the outer (the stress higher than 70% of the fatigue limit

Outcome

- Change from the safe-life concept to the fail-safe