Dynamická únosnost a životnost

Lekce 3
-
Stress-Based Fatigue Analysis, Part #3

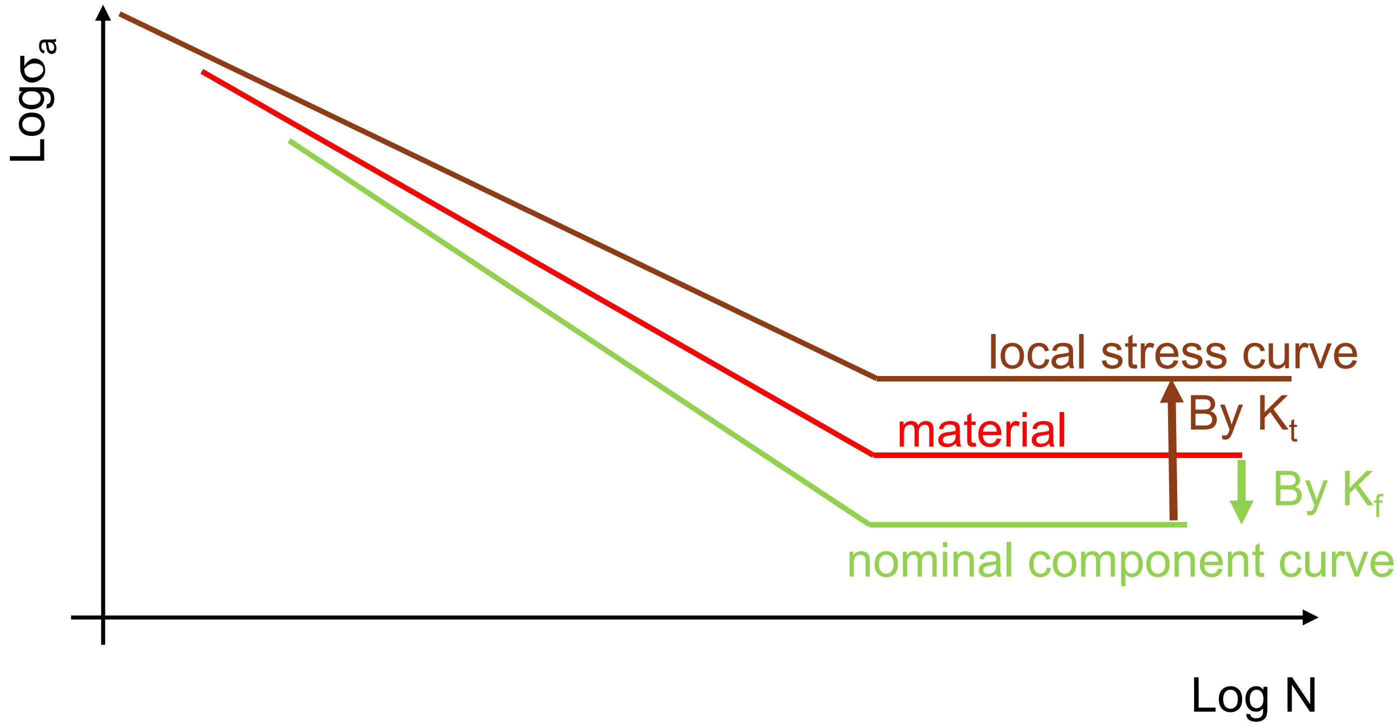
Jan Papuga

S-N Analysis

Have we discussed all effects?

Notch factor modification

The S-N curve has to be modified to cover the transformation: MATERIAL -> COMPONENT



Fatigue limit:

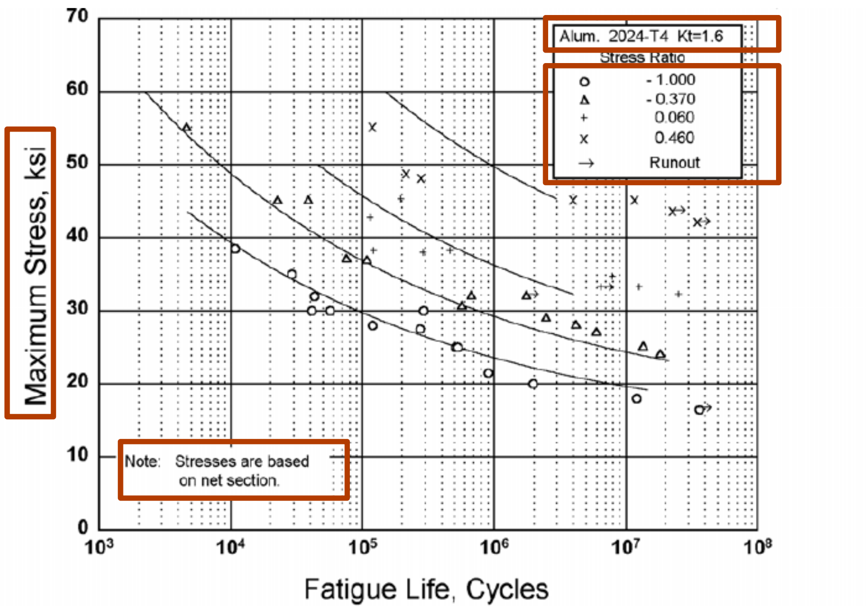
σ_{FL} , σ_C

$$K_f = \frac{\sigma_{FL,mat}}{\sigma_{FL,notched}} < K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

Other factors to face

$$\sigma_{FL,N} = \frac{\sigma_{FL} \cdot k_L \cdot k_{SF} \cdot k_S \cdot k_T}{K_f}$$

- ***k**$_L$* **– load factor**
- ***k**$_{SF}$* **– surface finish factor**
- ***k**$_S$* **– size factor**
- ***k**$_T$* **– thermomechanical treatment factor**



Product Form: Rolled bar, 1.125 inch diameter

Properties:	TUS, ksi	TYS, ksi	Temp., °F
	73	49	RT

Specimen Details: Semicircular
V-Groove, $K_t = 1.6$
0.450 inch gross diameter
0.400 inch net diameter
0.100 inch root radius, r
60° flank angle, ω

Surface Condition: As machined

Test Parameters:
Loading - Axial
Frequency - 1800 to 3600 cpm
Temperature - RT
Environment - Air

No. of Heats/Lots: Not specified

Equivalent Stress Equation:
Log N_f = 12.25-5.16 log (S_{eq}-18.7)
$S_{eq} = S_{max} (1\text{-}R)^{0.57}$
Std. Error of Estimate, Log (Life) = 0.414
Standard Deviation, Log (Life) = 0.989
R^2 = 82%

Here they are:

1. **Mean stress effect**
2. **Statistics**

Mean stress effect

Stress ratio

~ Coefficient of cycle asymmetry

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{\sigma_m - \sigma_a}{\sigma_m + \sigma_a}$$

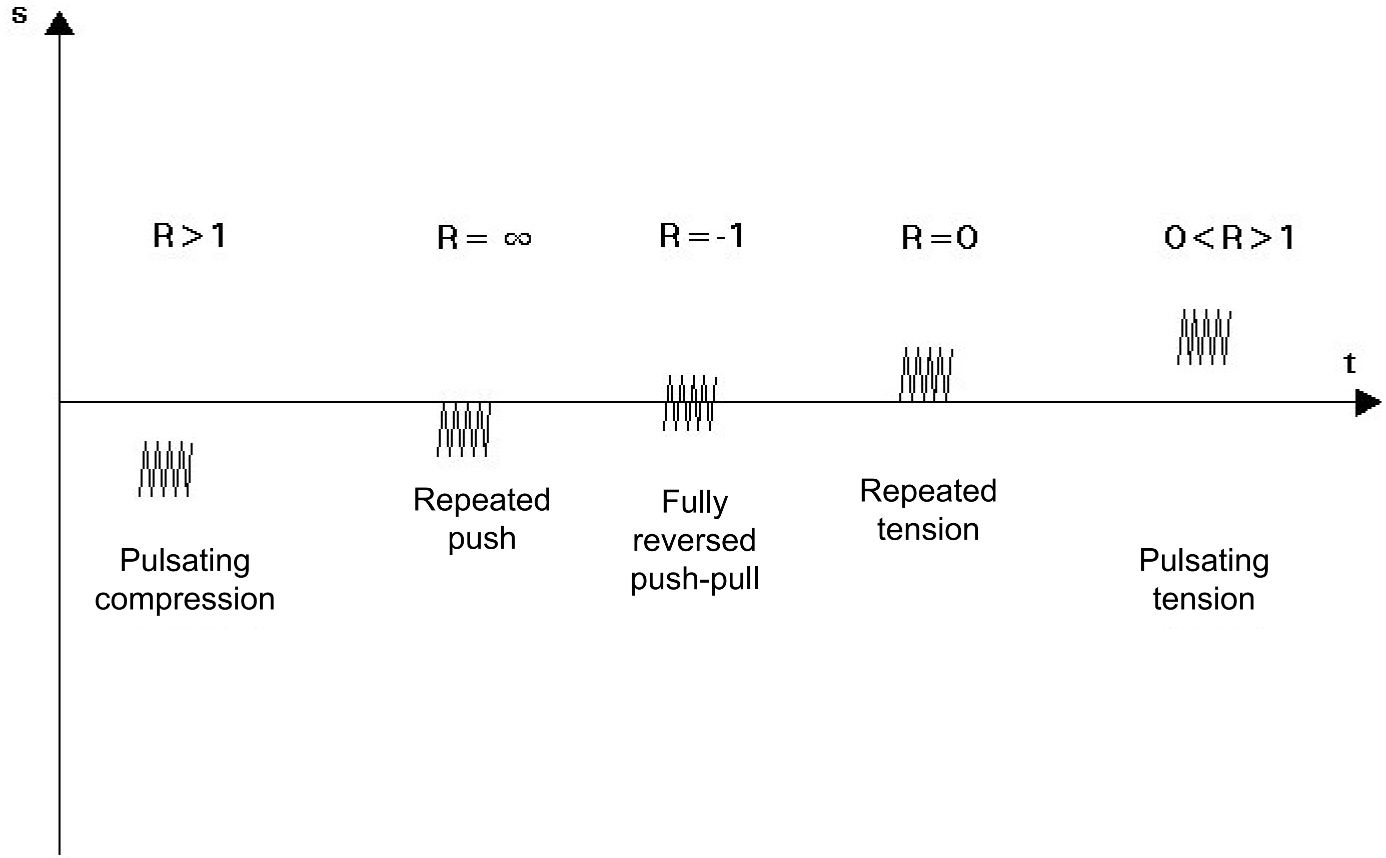
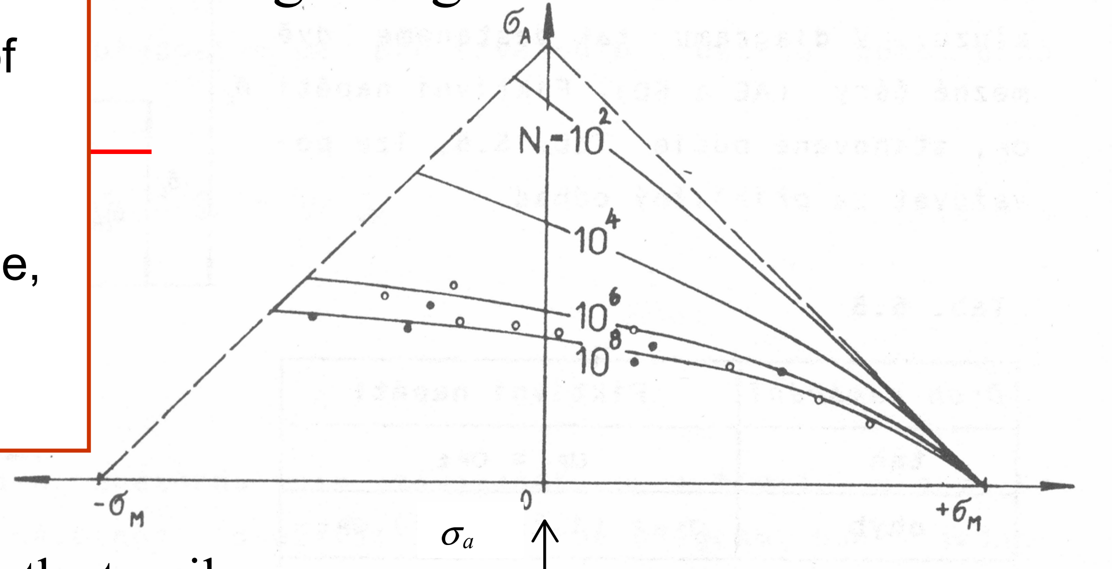


Figure 5 Stress cycles with different mean stresses and R-ratios.

Mean stress effect

Goodman, 1890:
".. whether the assumptions of the theory are justifiable or not.... We adopt it simply because it is the easiest to use, and for all practical purposes, represents Wöhler's data."

Haigh diagram

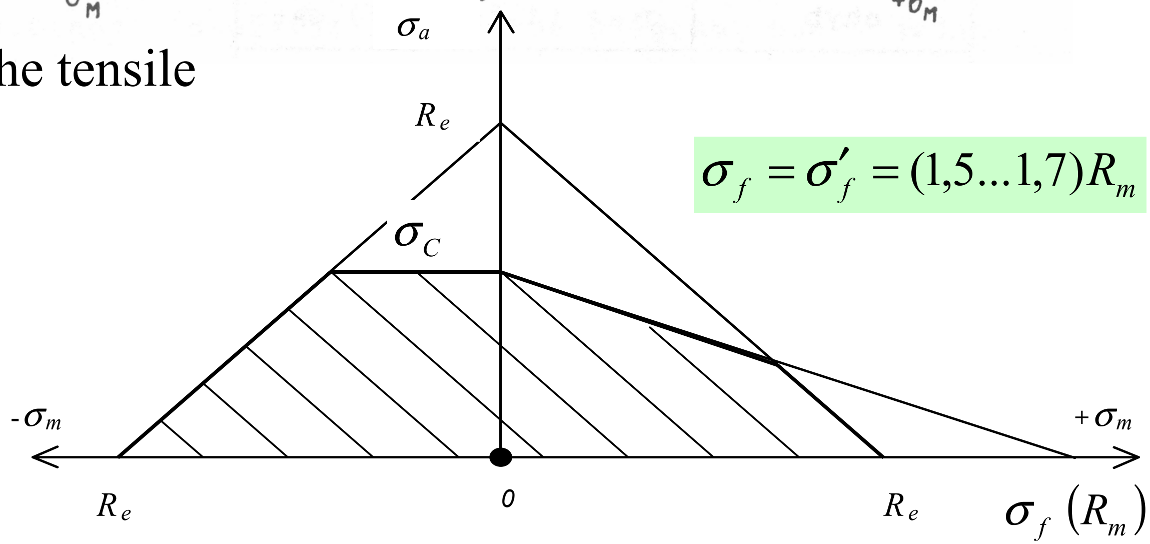


Fatigue strength reduction in the tensile area:

$$\sigma_a = \sigma_C \left[1 - \left(\frac{\sigma_m}{\sigma_f} \right)^m \right]$$

m=1 Goodman line

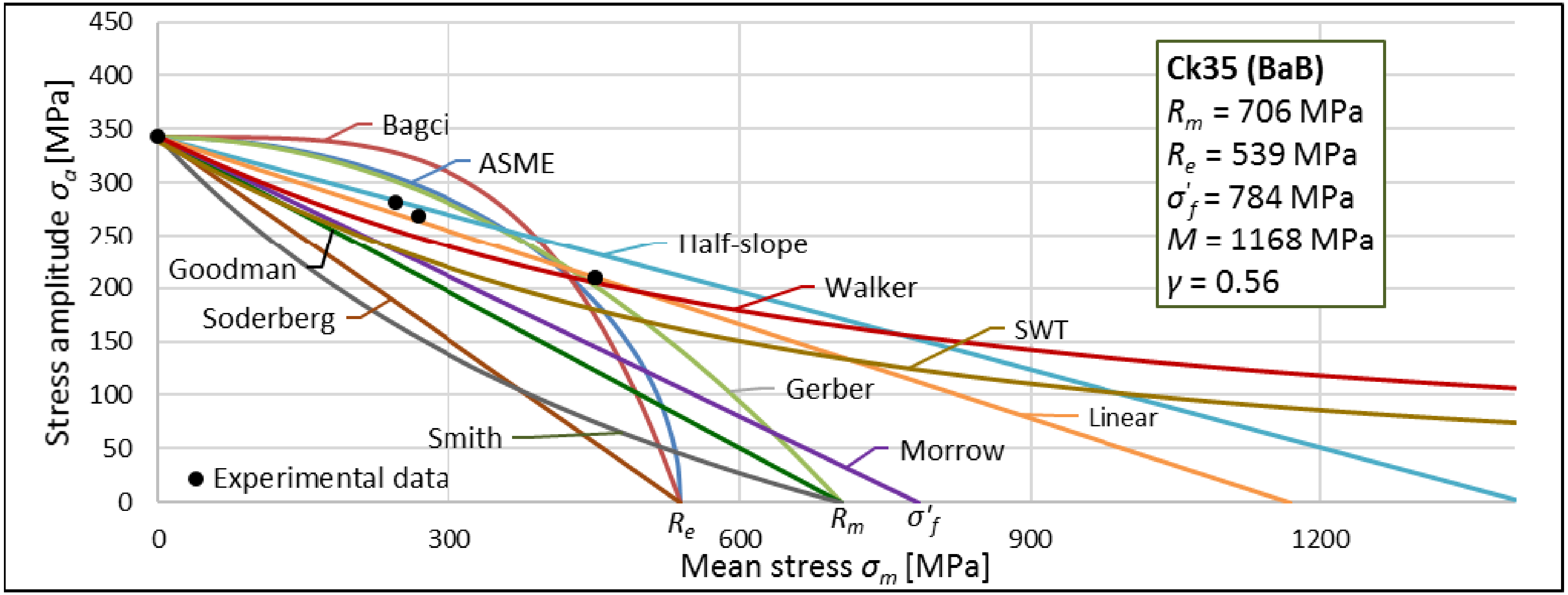
m=2 Gerber curve



$$\sigma_f = \sigma'_f = (1{,}5 \ldots 1{,}7) R_m$$

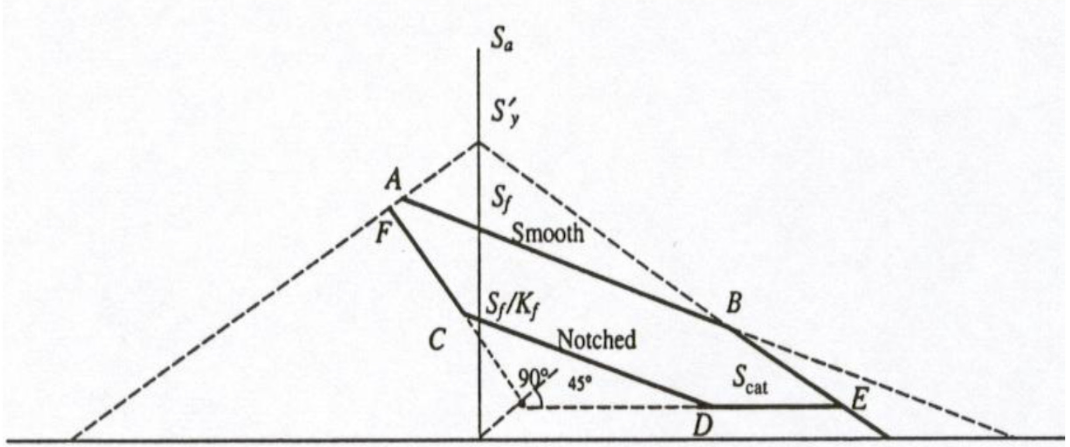
Haigh diagram

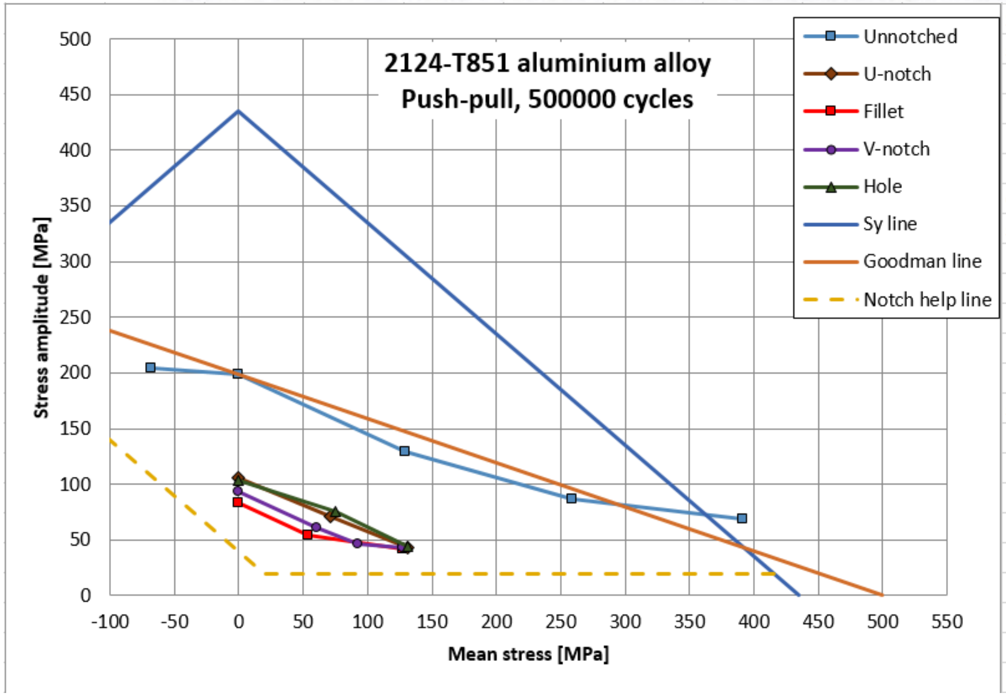
There are very many methods to estimate the Haigh diagram in the mean tensile region, see below:



Haigh diagram according to Fatemi

σ_f – true fracture stress
S_y – yield stress
S_f – fatigue limit
S'_y – cyclic yield stress
S_{cat} – critical alternating tensile stress (below this stress the cracks will not propagate)
= 70 MPa (hard steel)
= 30 MPa (mild steel)
= 20 MPa (high-strength aluminum)





Sy line	
sigm	siga
435	0
0	435
-435	0

Goodman line	
sigm	siga
500	0
0	198.3
-100	237.96

Notch help line	
sigm	siga
415	20
20	20
-130	170

Goodman method

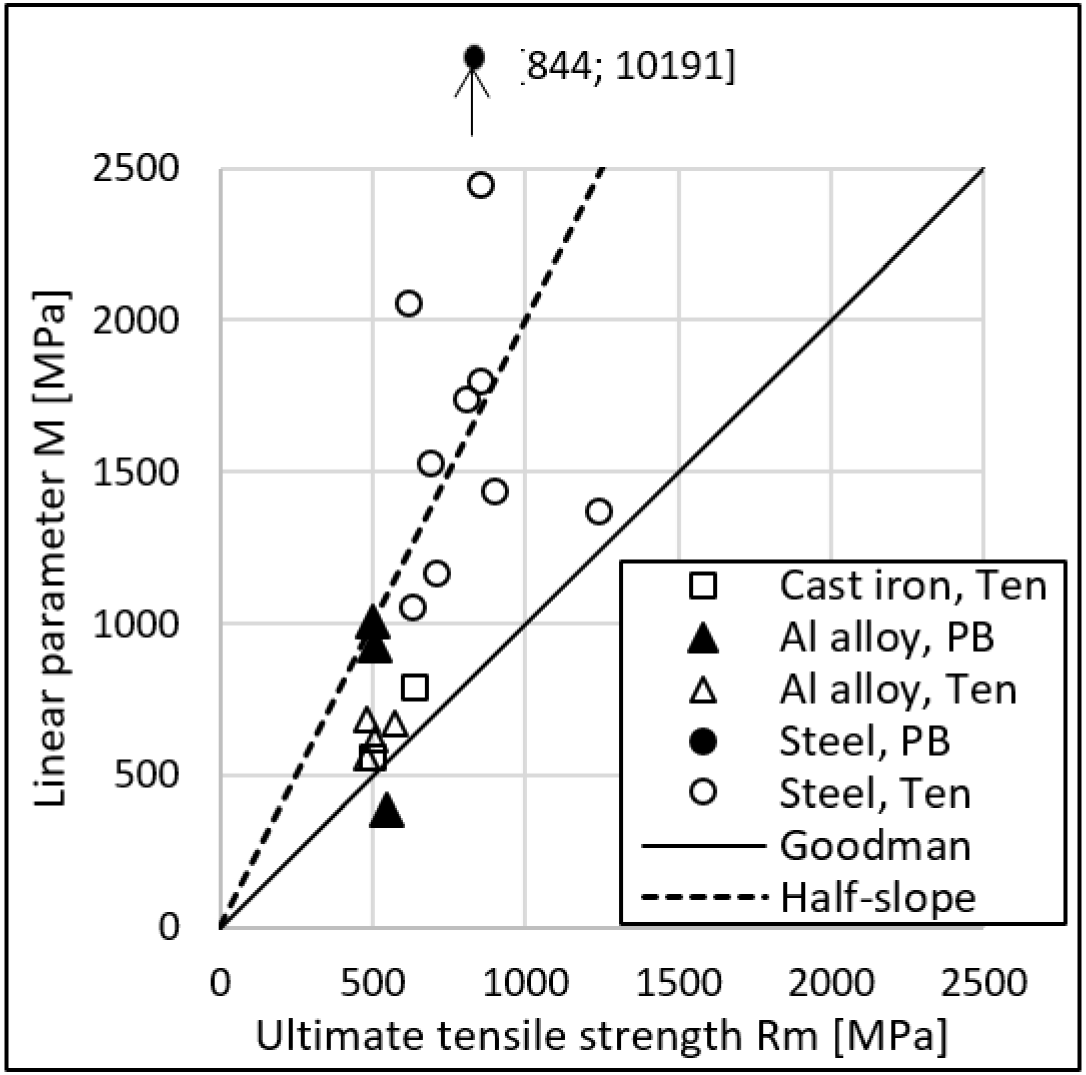
Generalized solution

$$\sigma_a = \sigma_C \left[1 - \left(\frac{\sigma_m}{\cancel{\sigma_f}} \right)^m \right]$$

1

M

- Linear optimization used to obtain the best fit *M* parameters
- Results:
  - Half-slope method (*M*=2 *R*$_m$) provides generally better prediction, but can get unsafe (the case of hardened 4130 steel)



Mean stress sensitivity M

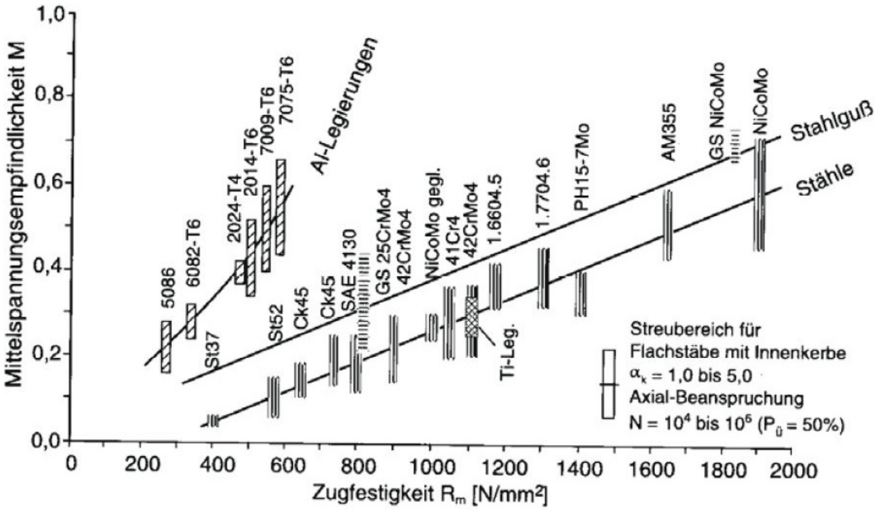


Bild 2-1: Mittelspannungsempfindlichkeit verschiedener Werkstoffe in Abhängigkeit von der Zugfestigkeit /5/ (aus /25/).

$$M = \frac{\sigma_{a,R=-1}}{\sigma_{a,R=0}} - 1$$

Anon.: Mittelspannungseinfluß auf das Schwingfestigkeitsverhalten geschweißter Aluminiumlegierungen. [AIF-Nr. 12676 N, DVS-Nr. 9.031]. TU Braunschweig, Braunschweig 2004.

Haibach, E.: Betriebsfestigkeit – Verfahren und Daten zur Bauteilberechnung. Springer 2006.

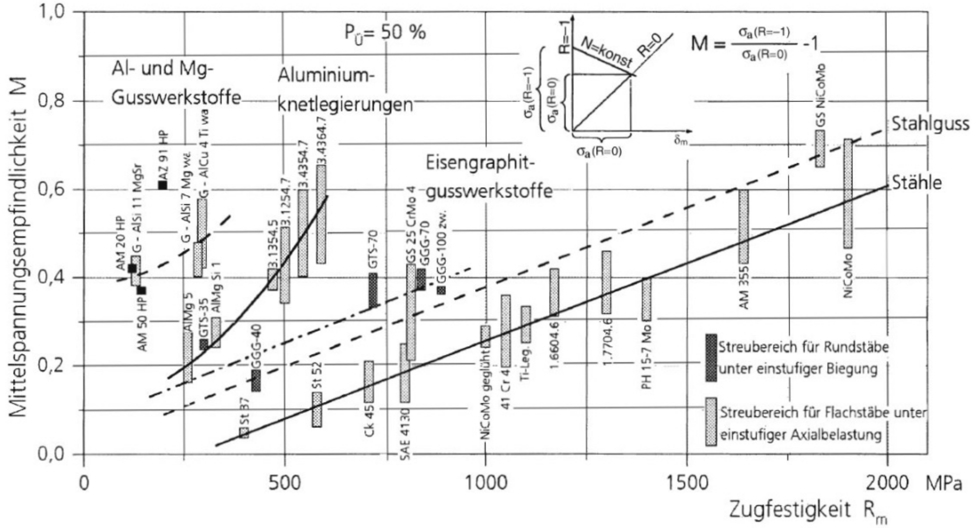
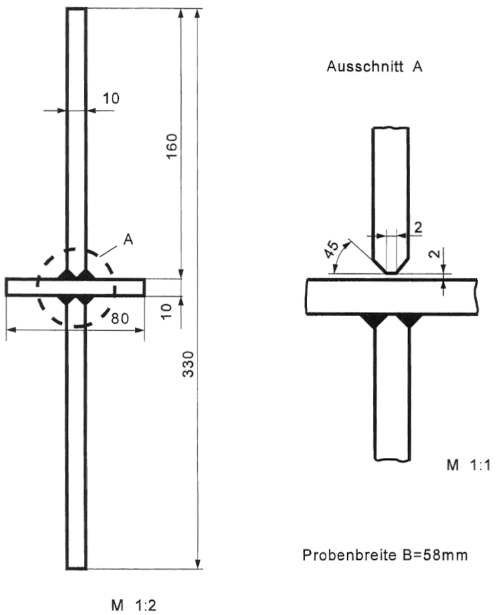
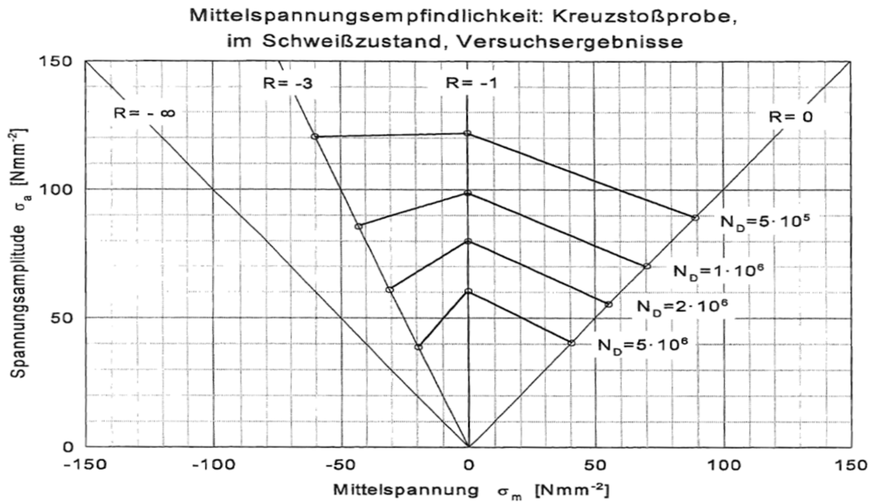


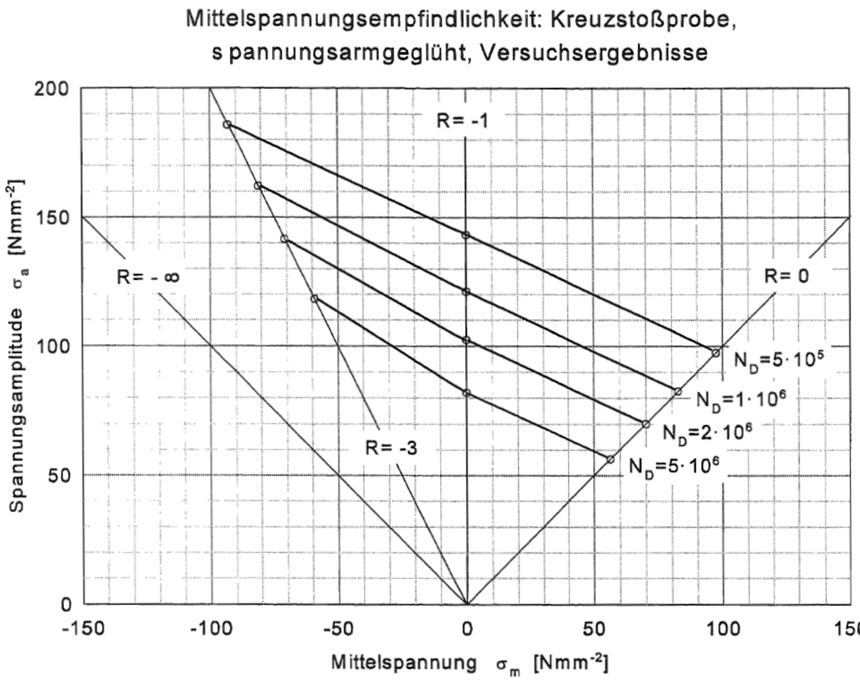
Abb. 2.1–9. Mittelspannungsempfindlichkeit *M* verschiedener Stahl-, Eisenguss und Aluminium-Werkstoffe, nach Sonsino

Compressive mean stress – exceptions?



Bakczewitz, F.; Friedrich, P.; Naubereit, H.: Zum Einfluss der Druckmittelspannung auf die Betriebsfestigkeit. Universität Rostock 1998.





Safety coefficient against break

Depends on fixed parameters of the loading

- Here for R = const:

$$\frac{\sigma_A}{\sigma_C}+\frac{\sigma_M}{\sigma_f}=1 \qquad (1)$$

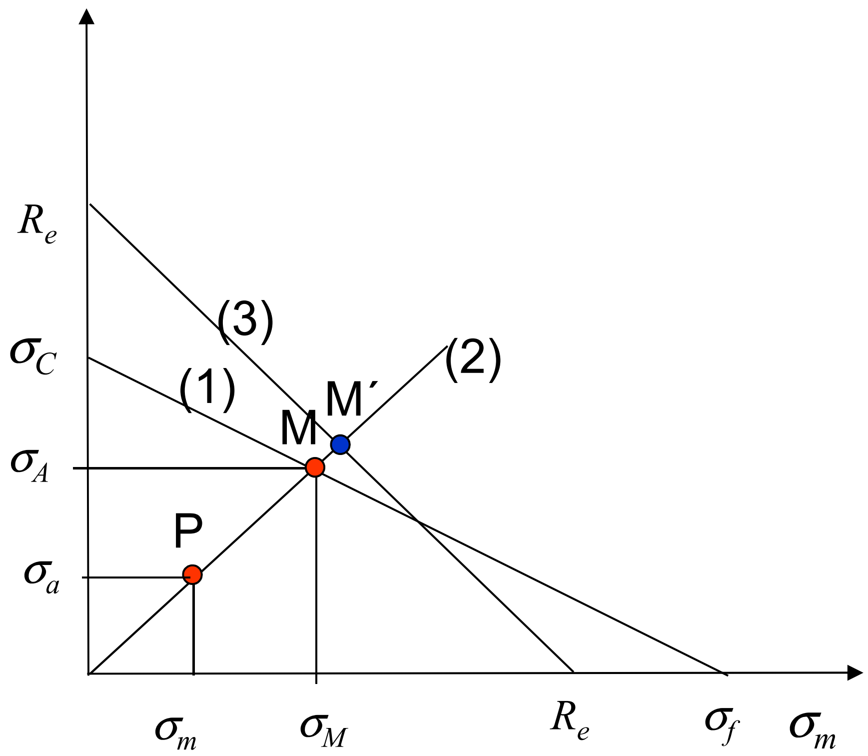
$$\sigma_A = k_C \cdot \sigma_a \qquad (2)$$

$$\sigma_M = k_C \cdot \sigma m$$

$$\frac{1}{k_C}=\frac{1}{\frac{\sigma_C}{\sigma_a}}+\frac{1}{\frac{\sigma_f}{\sigma_m}}=\frac{1}{k_a}+\frac{1}{k_m}$$

$$\frac{1}{k_{\mathrm{Re}}}=\frac{1}{\frac{R_e}{\sigma_a}}+\frac{1}{\frac{R_e}{\sigma_m}} \qquad (3)$$

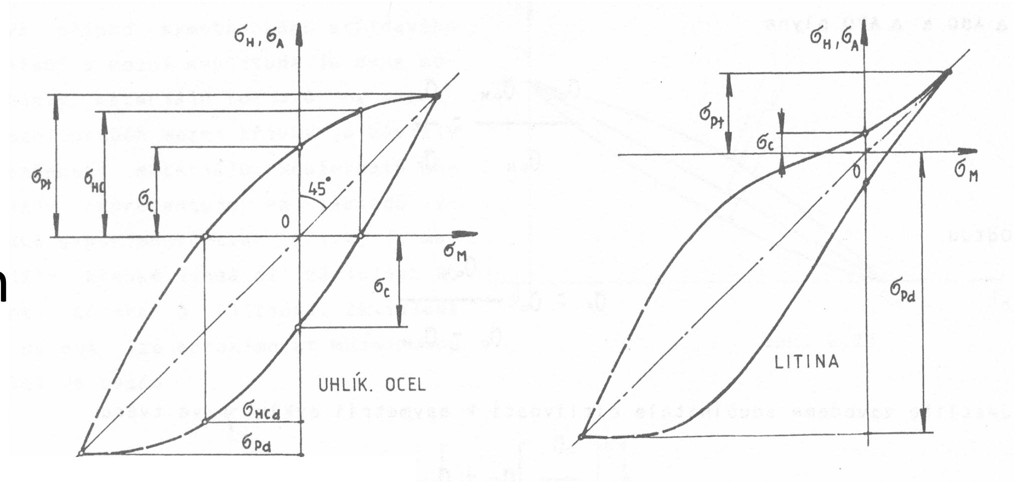
$$k=\min\left(k_C,k_{\mathrm{Re}}\right)$$

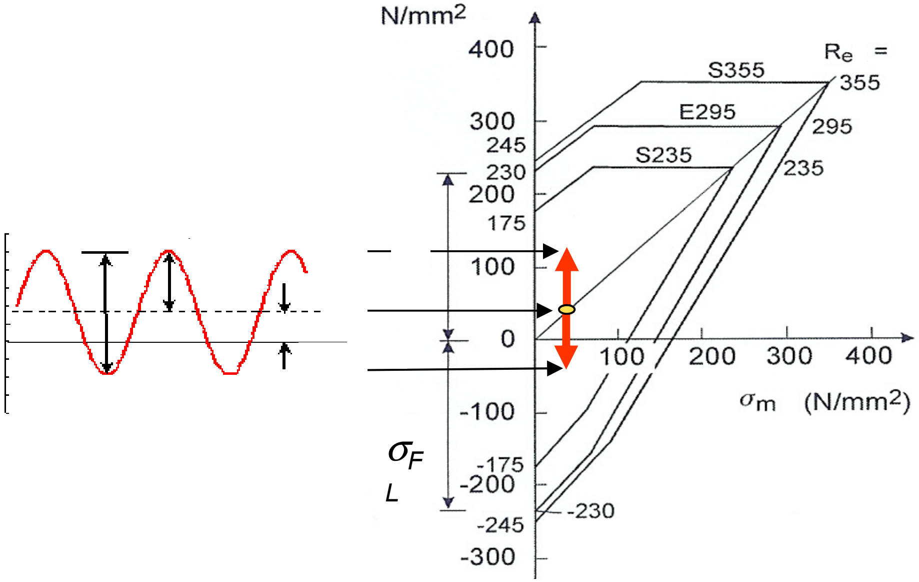


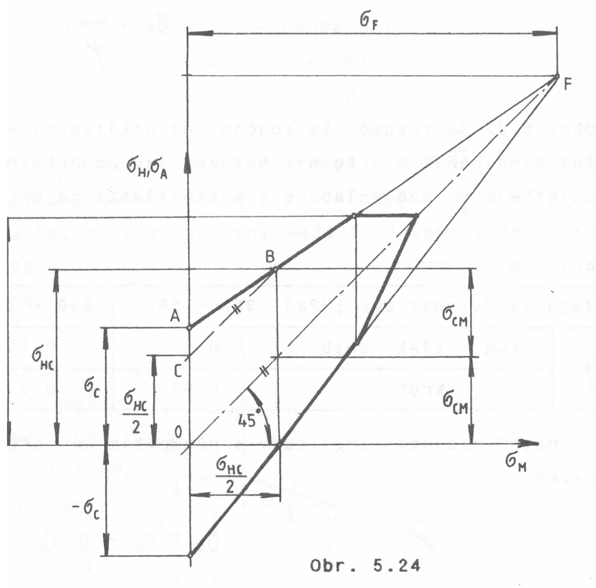
Smith’s diagram

Typically used in railway industry today

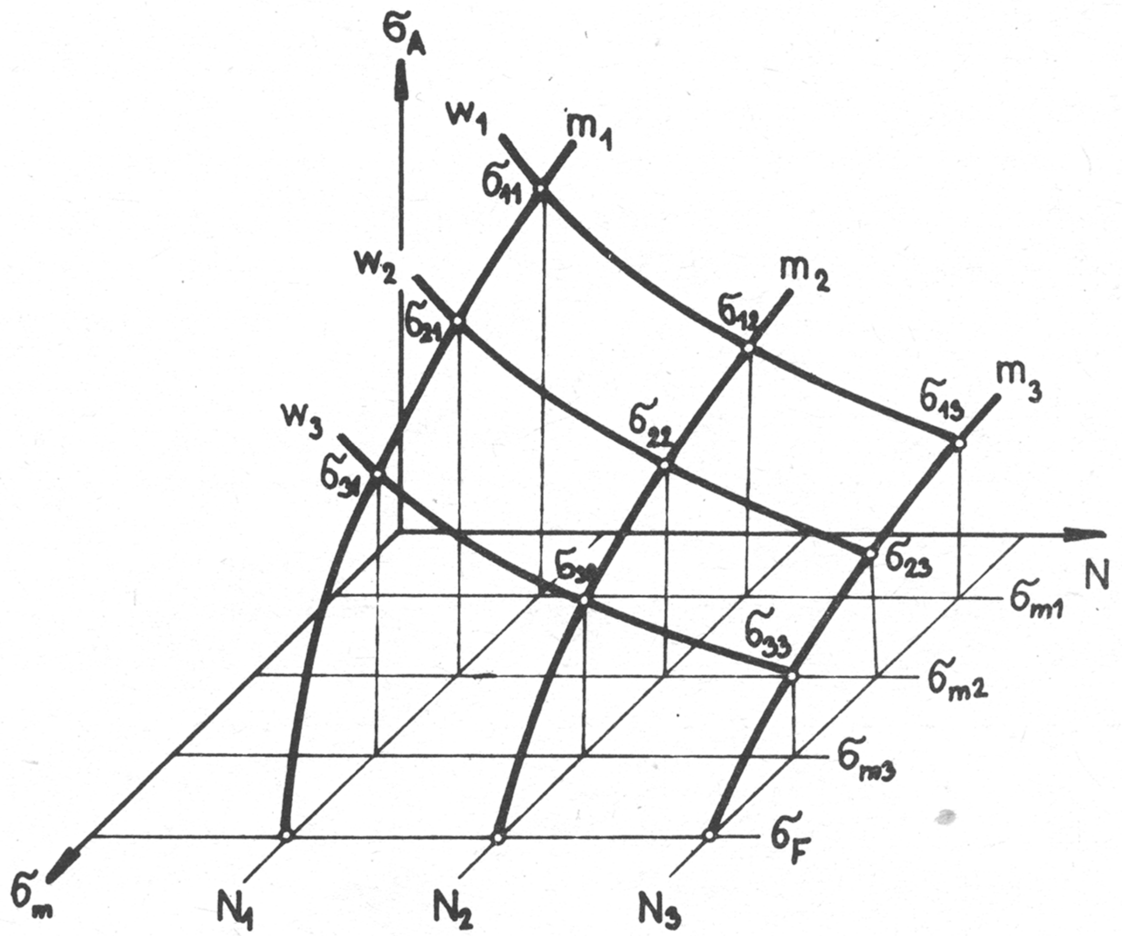
Useful for graphical evaluation above all



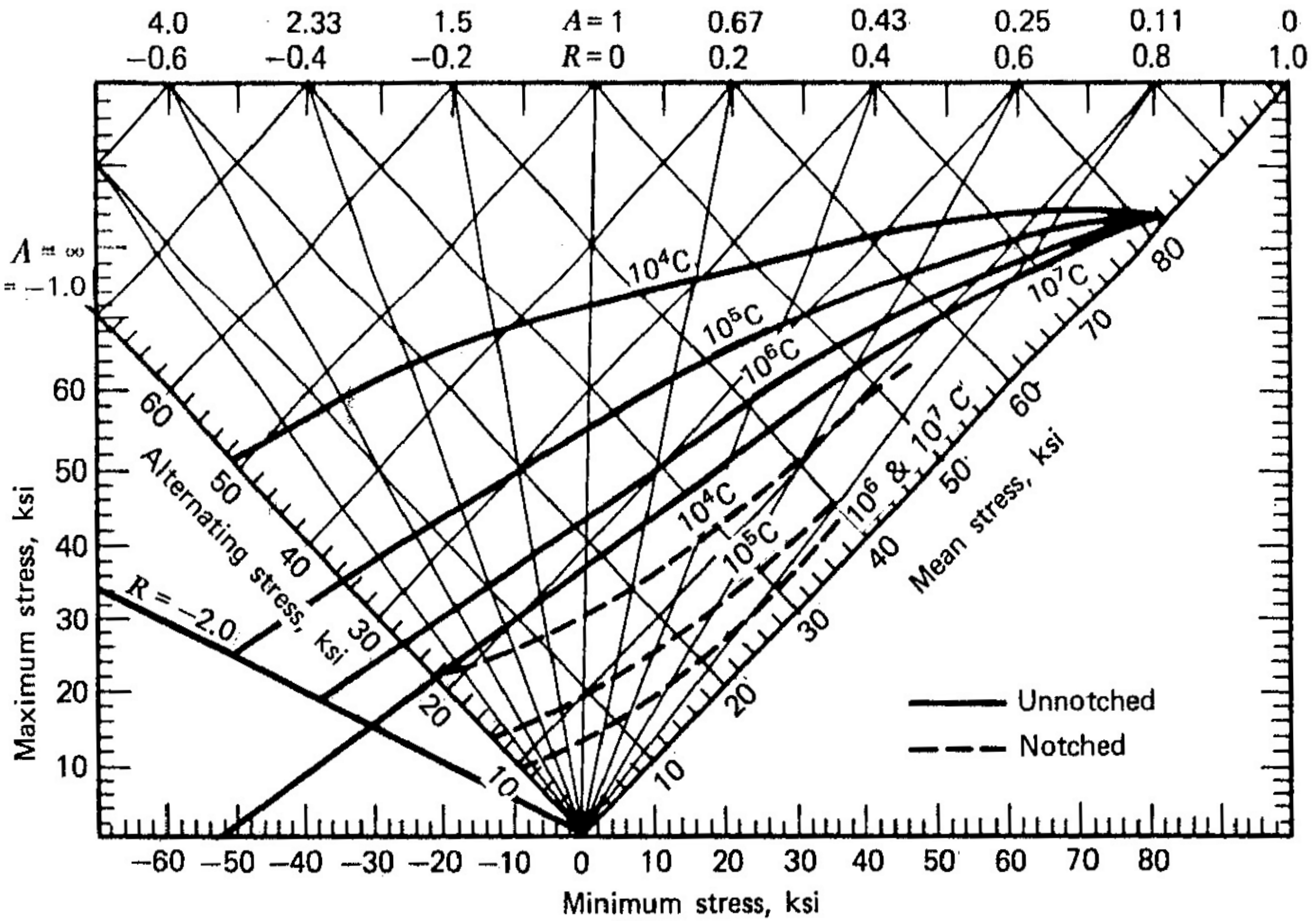




Mean stress effect – 3D diagram



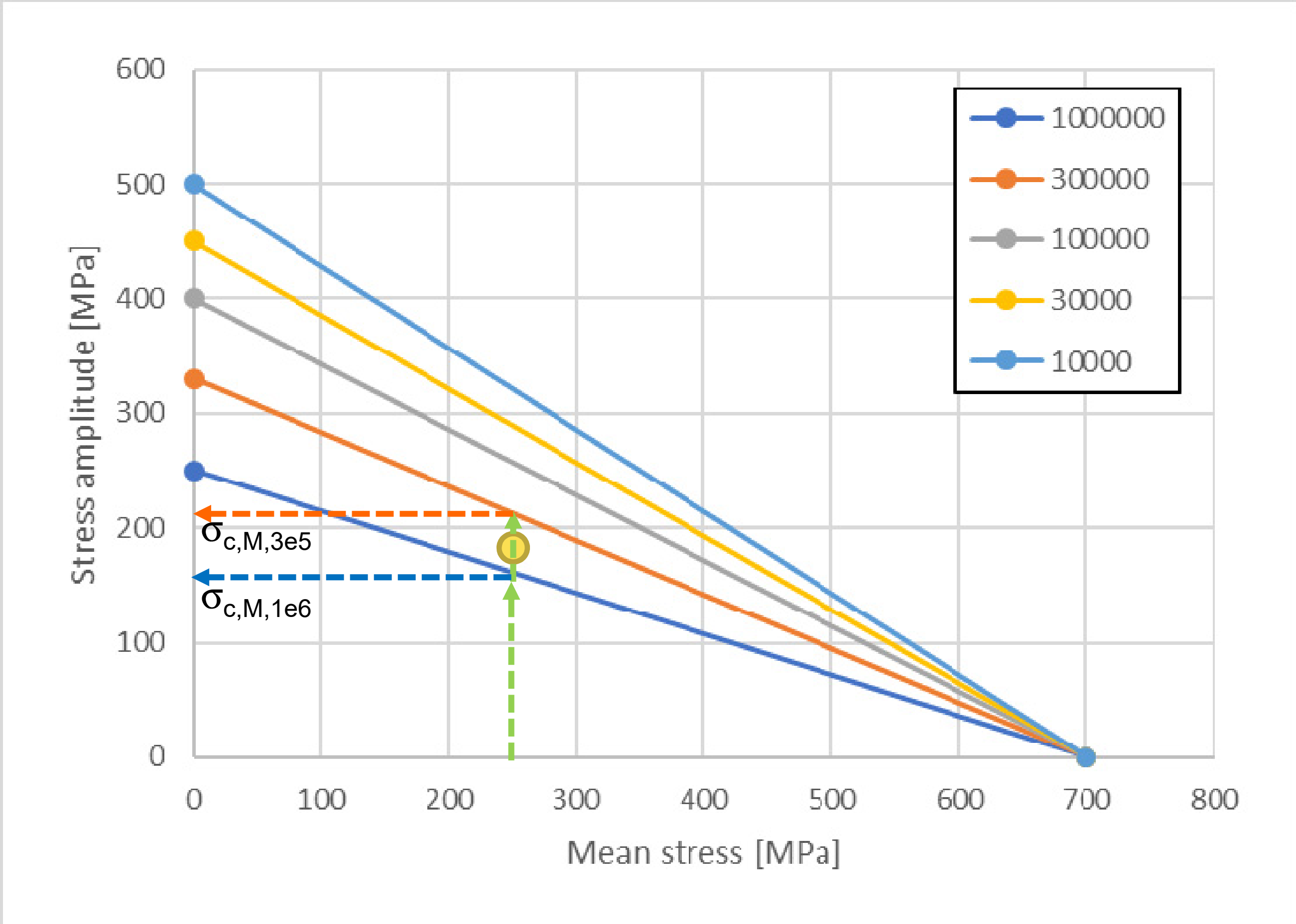
Mean stress effect – 2D diagram



Mean stress effect - reduced fatigue limit

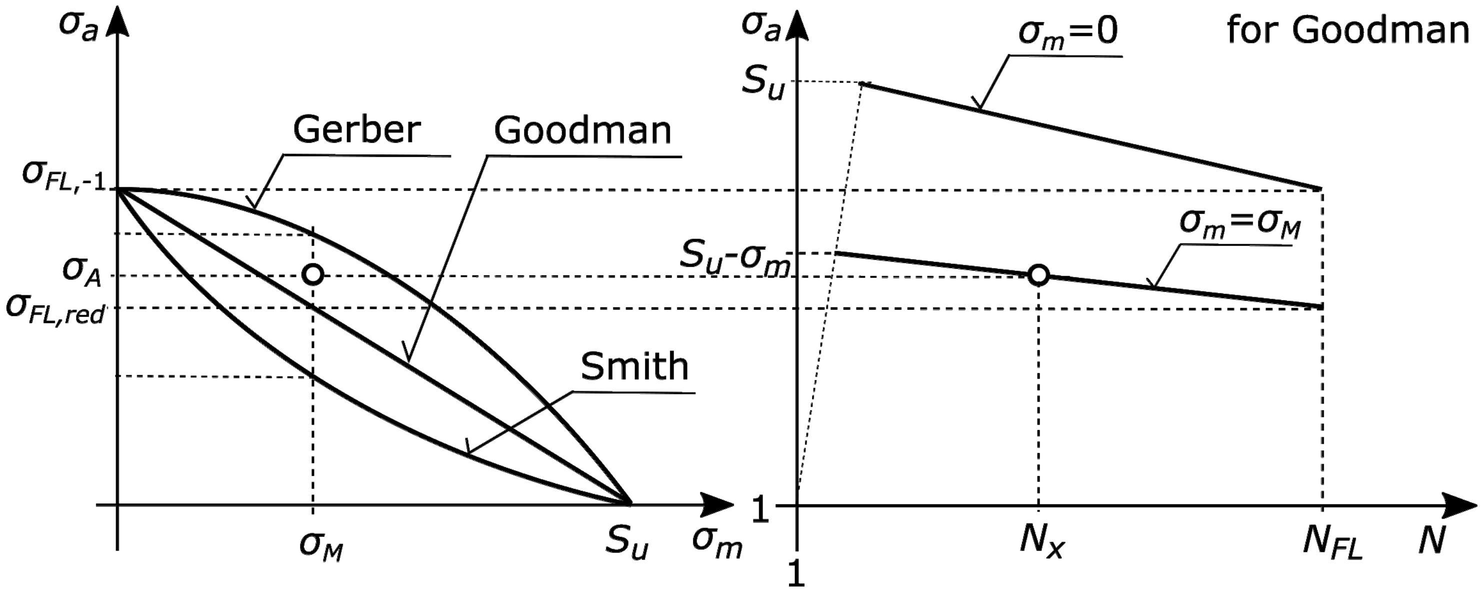
Haigh diagram shows, how the fatigue limit in fully reversed loading is reduced to the fatigue limit at given mean stress

Another step must be taken to retrieve the relevant lifetime - see next slides



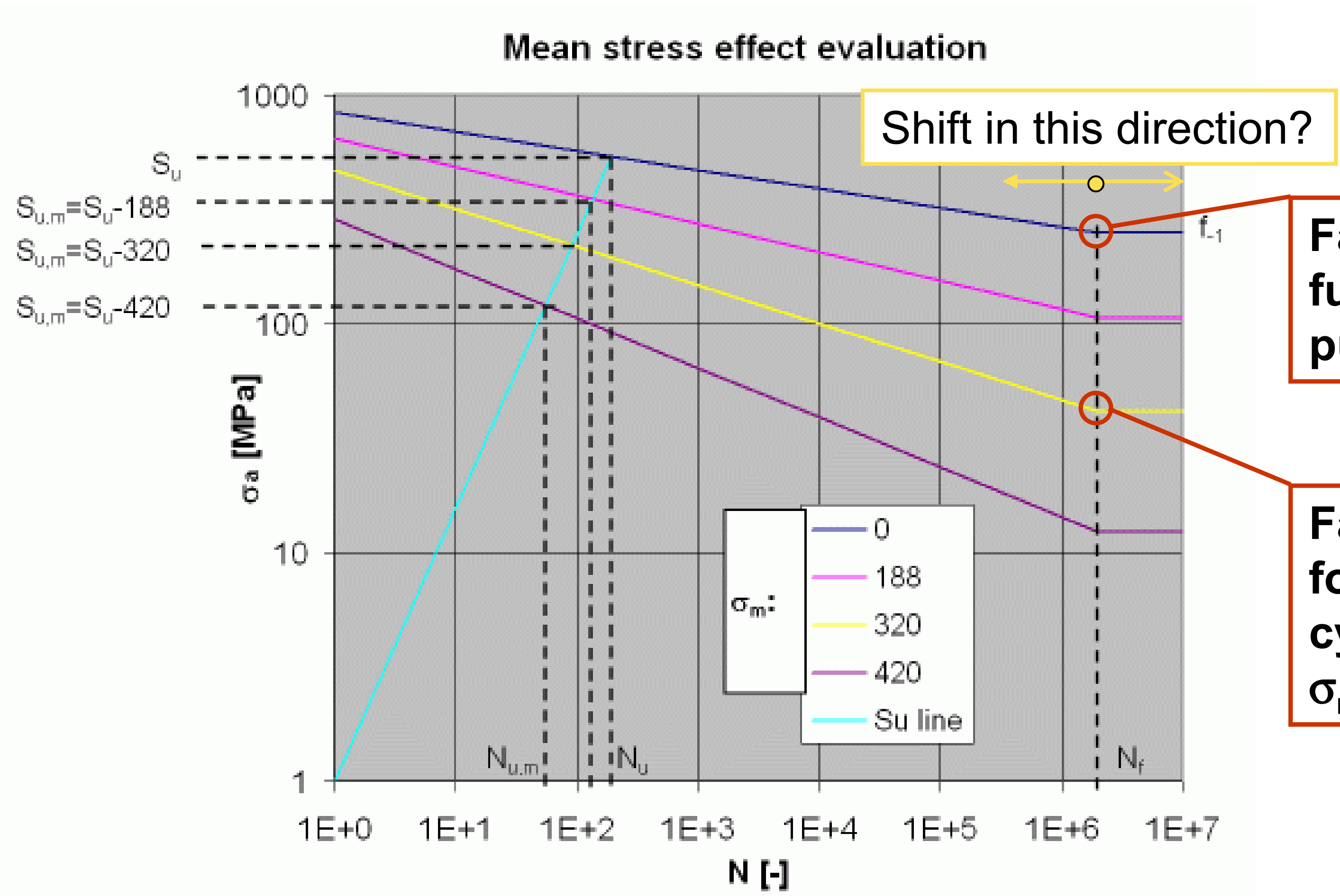
Reduced fatigue limit

1. Modify the fatigue limit by mean stress
2. Find the second point of the S-N curve in the quasi-static region
3. Construct the S-N curve related to the given mean stress
4. Retrieve the lifetime from this curve while inputing the stress amplitude



Mean stress effect in the S-N curve

Whole S-N curve has to be modified, not only FL:



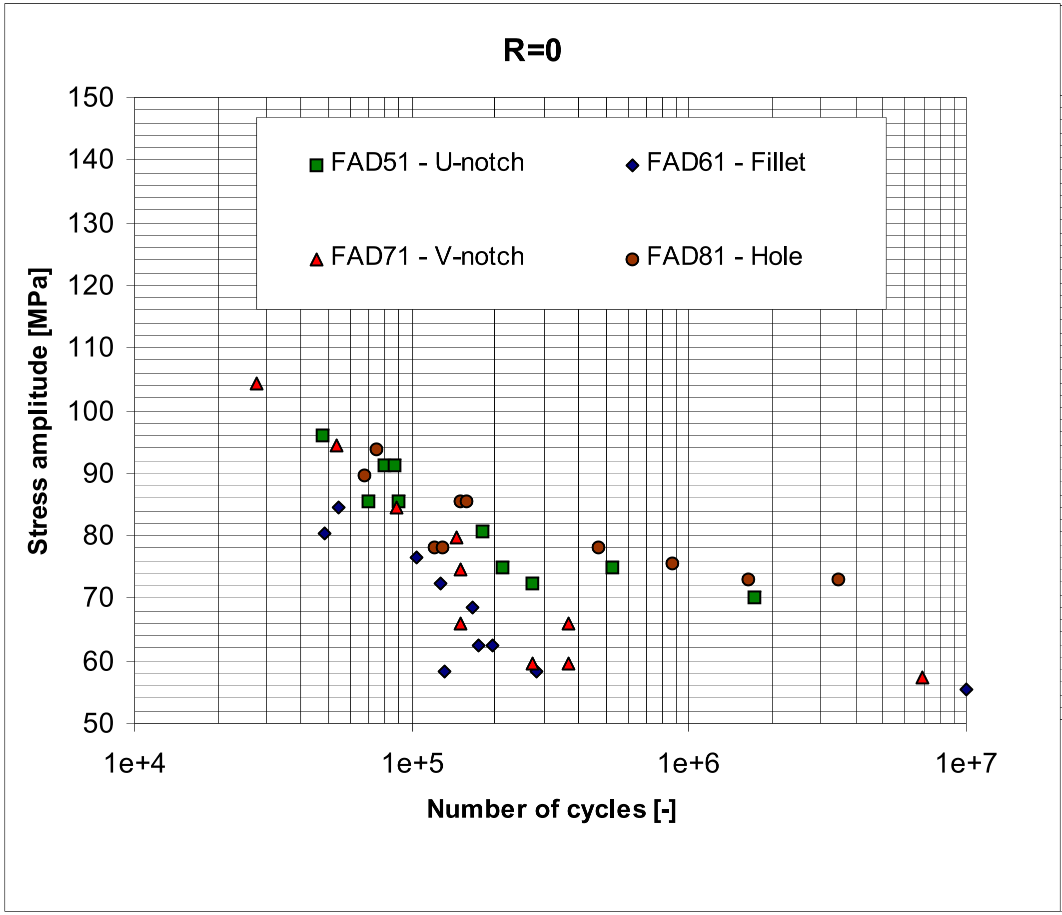
Fatigue limit in fully reversed push-pull

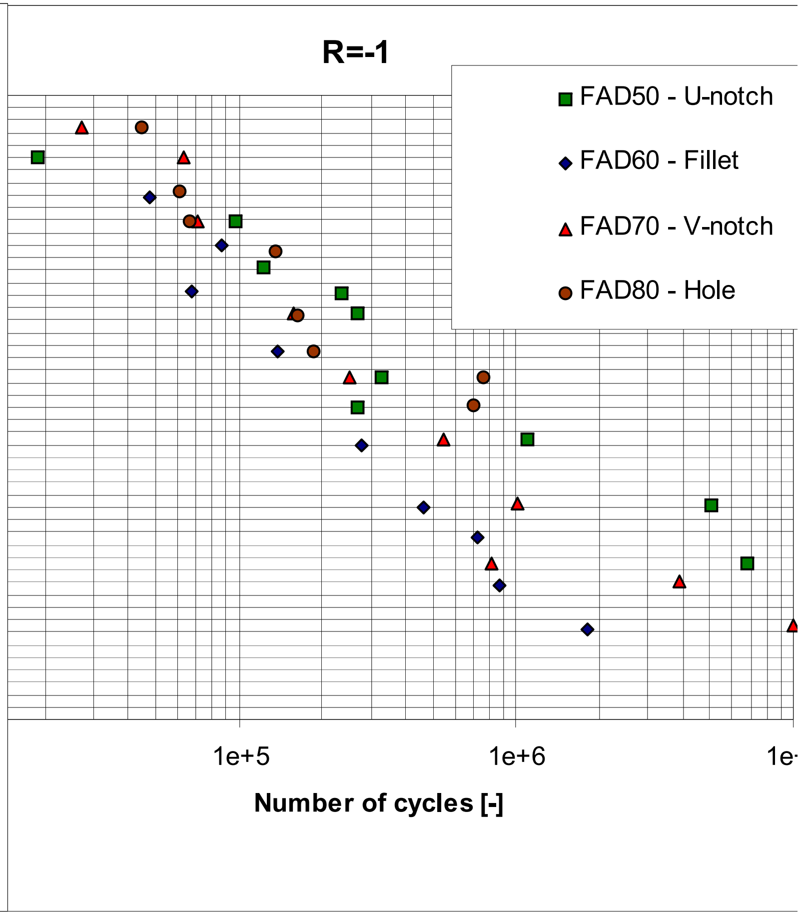
Fatigue limit for a stress cycle with σ_m=320MPa

Fatigue limit position

Fillet r=0.4 mm
U-notch r=1.6 mm
V-notch r=1.6 mm
Hole r=2 mm

Aluminium alloy 2124T851
Kt = 2.3

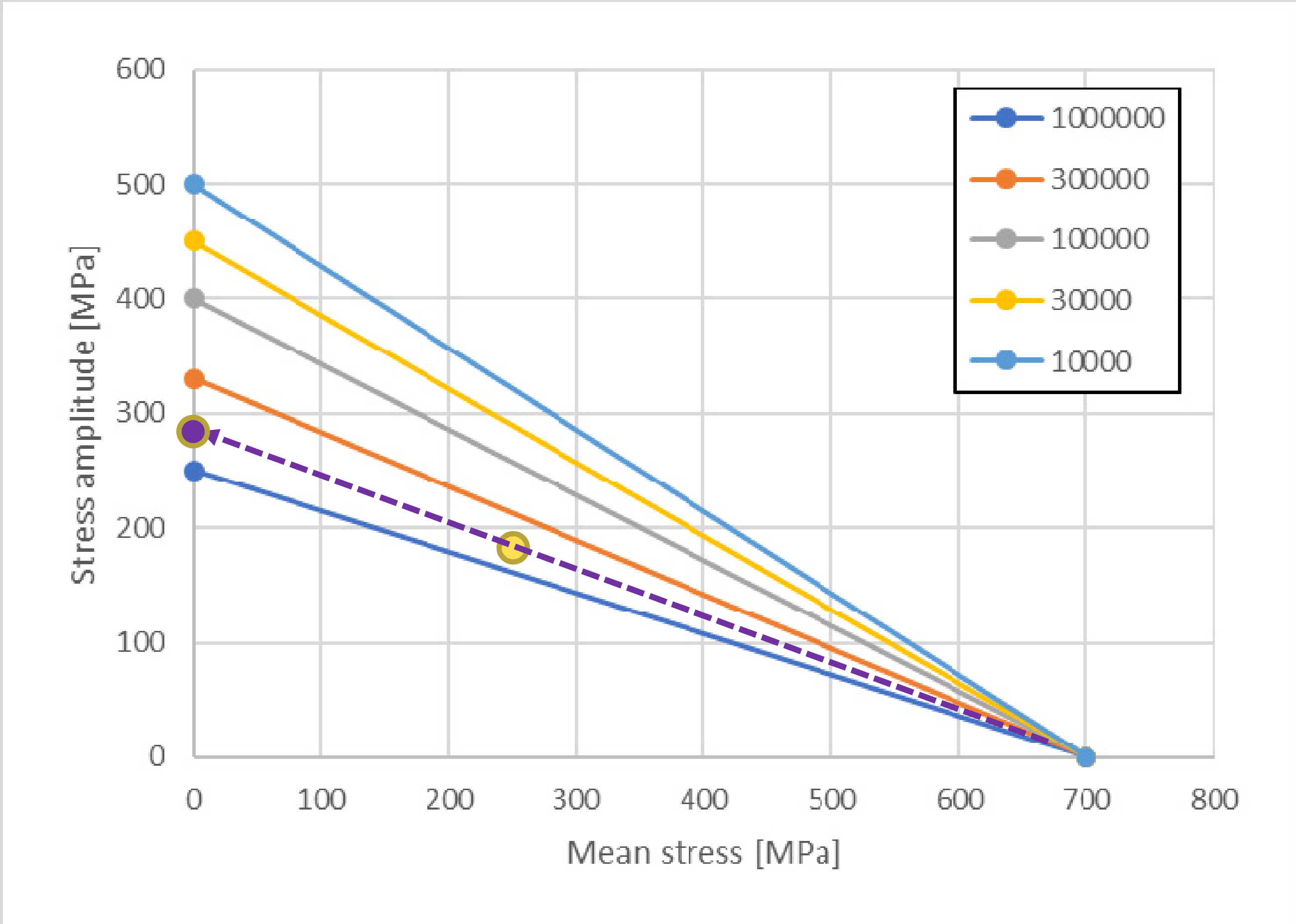




MSE – equivalent stress amplitude

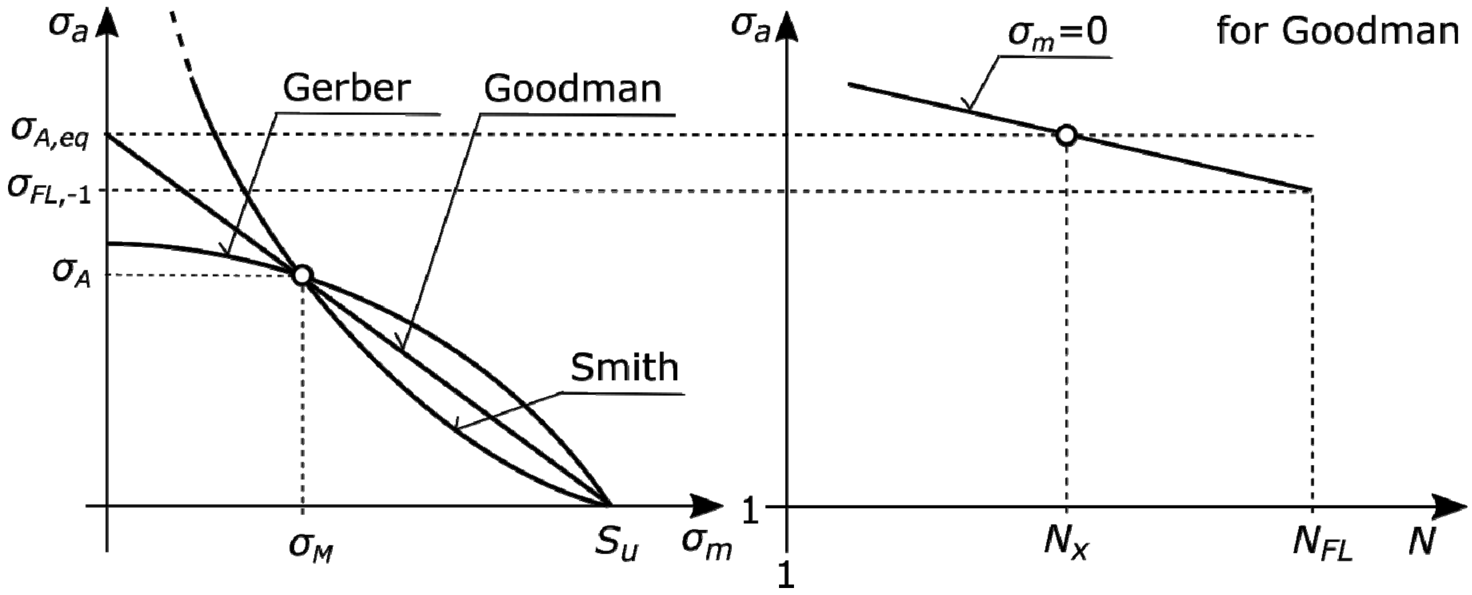
By using the same law (Goodman line here), the equivalent stress amplitude is computed

This amplitude is then used with the S-N curve in fully reversed loading to retrieve the lifetime

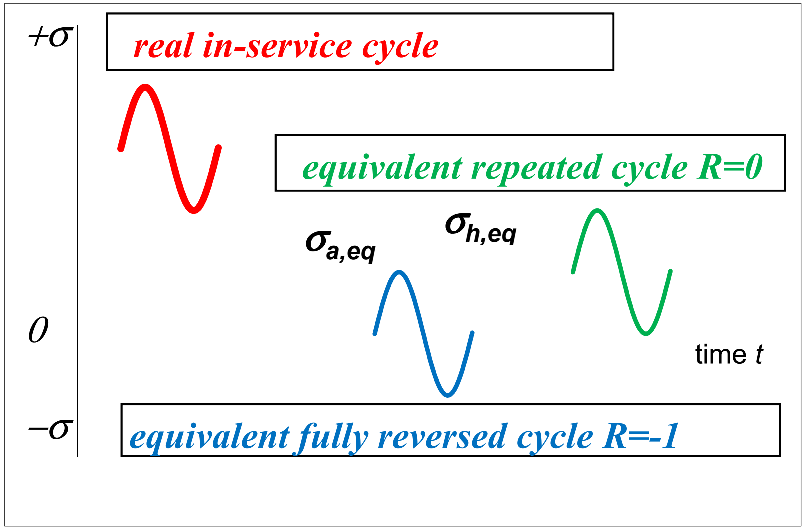


Equivalent stress amplitude

1. Find the equivalent stress amplitude
2. Use it with the S-N curve of $\sigma_m = 0$ MPa



Equivalent stress cycle



= the general cycle is transformed into a cycle with other mean stress (repeated or reversed cycle) with equal damaging effect

Landgraf and Morrow:

$$\sigma_{a,eq} = \frac{\sigma_a}{1-\left(\frac{\sigma_m}{\sigma'_f}\right)}$$

Oding / Walker:

$$\sigma_{h,eq} = \sigma_{\max}\cdot(1-R)^w = 2^w \cdot \sigma_{\max}^{1-w}\cdot\sigma_a^w$$

SWT parameter:

$$\sigma_{a,eq} = \sqrt{(\sigma_a+\sigma_m)\cdot E\varepsilon_a}, \quad pro\ \sigma_m > 0$$

$$\sigma_{a,eq} = \sigma_a, \quad pro\ \sigma_m \le 0$$

MIL HDBK:

$$\sigma_{h,eq} = \sqrt{2\cdot(\sigma_a+\sigma_m)\cdot\sigma_a}, \quad for\ \sigma_m > 0$$

$$\sigma_{h,eq} = \sqrt{2}\cdot\sigma_a, \quad for\ \sigma_m \le 0$$

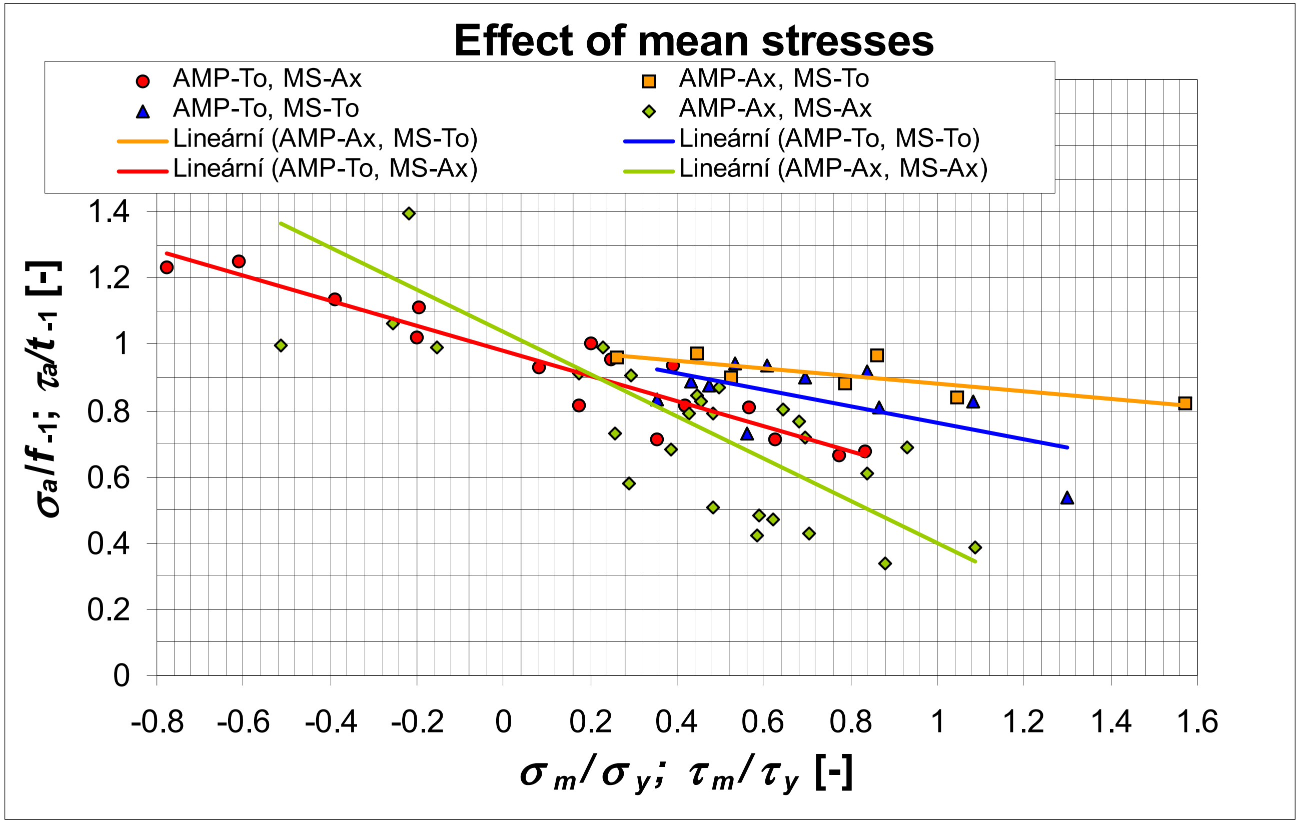
Equals to Oding when w=0.5

„Walker“:

$$\sigma_{a,eq} = \sigma_a^{\gamma}\cdot(\sigma_m+\sigma_a)^{1-\gamma}$$

Other load modes

NOTE: The statement that the mean shear stress has no effect on fatigue limit is untrue.



Mean stress effect - summary

Tensile mean stress decreases fatigue strength or lifetime

- Has to be covered

Compression mean stress

can increase fatigue strength or lifetime

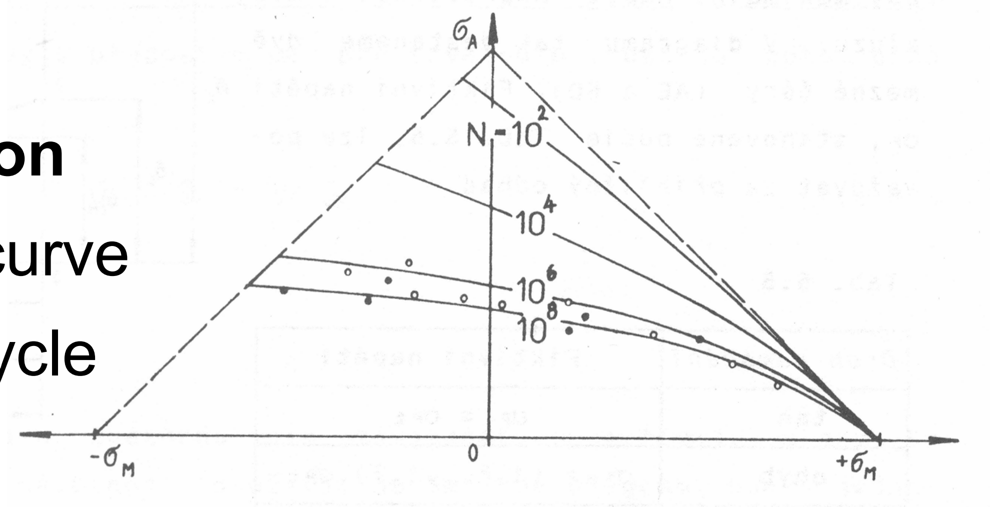
- Safe to neglect

Mean torsion stress also decreases fatigue strength or lifetime

- Has to be covered

Mean stress effect inclusion

- Modifying the fatigue curve
- Modifying the stress cycle



Statistical effect

will be presented by Prof. Růžička on November 30

Safety factor

To differentiate:

- On stresses
- On lifetime

FKM:

Table 4.5.1 Safety factors for steel *[3] (not for GS) and for ductile wrought aluminum alloys (A≥ 12,5 %).

j_D		Consequences of failure	
		severe	moderate ⟡1
regular inspections	no	**1,5**	1,3
	yes⟡2	1,35	1,2

⟡1 Moderate consequences of failure of a less important component in the sense of "non catastrophic" effects of a failure; for example because of a load redistribution towards other members of a statical indeterminate system. Reduction by about 15 %.

⟡2 Regular inspection in the sense of damage monitoring. Reduction by about 10 %.

Anon: Fatigue, Fail-Safe, And Damage Tolerance Evaluation of Metallic Structure for Normal Utility, Acrobratic, And Commuter Category Airplanes. [Advisory Circular AC 23-13A]. U.S. Government Printing Office, Washington 2005.

2-26. What scatter factors do I use in a fatigue analysis?

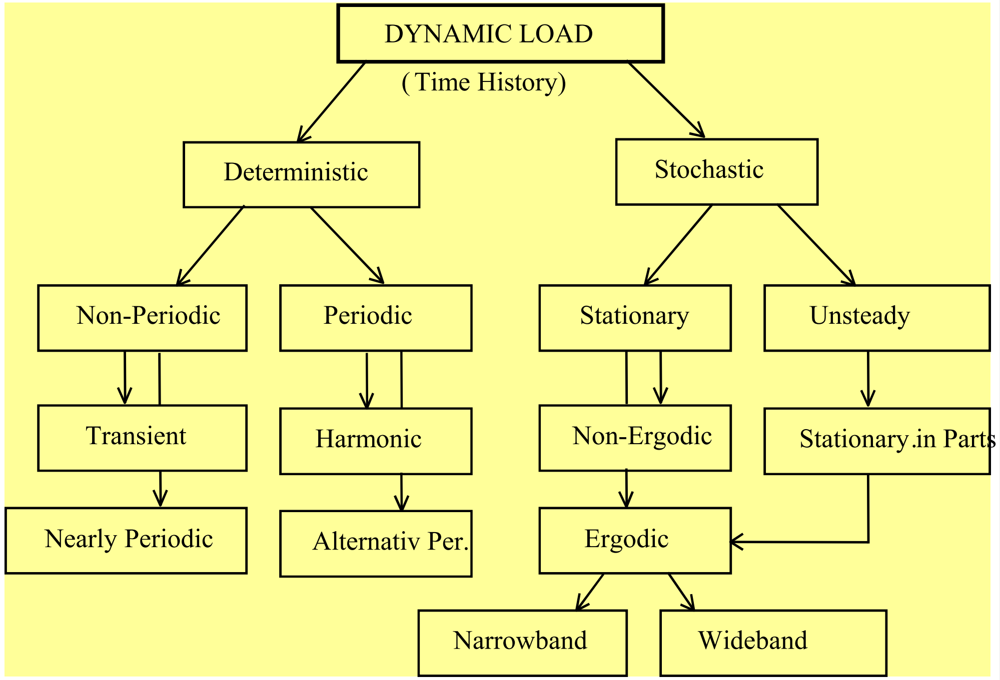
The scatter factor you use in a fatigue analysis is larger than the scatter factors you would apply to full-scale and component level test results. This is due to the uncertainties in a fatigue analysis outlined in the previous paragraph. The scatter factor used in an analysis depends on the type of metal, statistical basis, and applicability of the S-N data used in the analysis.

a. For aluminum structures, you may use the following:

(1) You may use the S-N data provided in Appendix 2. This S-N data is applicable to conventional built-up aluminum structure with no fittings (other than continuous splice fittings), no parts with high residual stresses, no unique structural features, and no stress concentrations greater than $K_t = 4$. This data is carried over from the guidance provided in Reference 3. Use of this S-N data, combined with a scatter factor of 8.0, has provided satisfactory service experience.

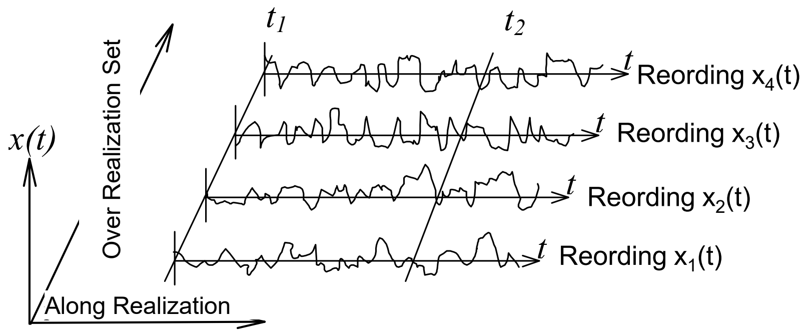
Load Analysis

Dynamic loading



Load type	Occurrence in real service
Harmonic loading with constant amplitude	**2%**
Triangle or trapezoidal cycle form with constant amplitude	**5%**
Block loading with constant amplitude in each block	**12%**
Stationary random loading	**9%**
Non-stationary random loading	**13%**
Process stationary just in parts	**41%**
Transient loads	**13%**
Other or special processes	**5%**

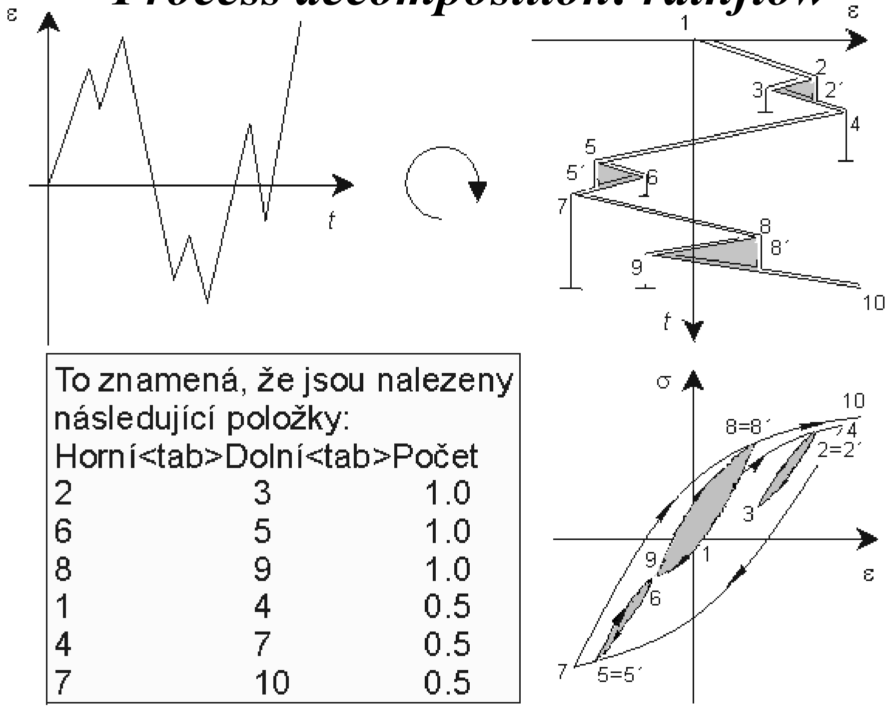
Bily M, Tydlacka V. Operating conditions as input information for fatigue life estimation. In: Measurement and Fatigue - EIS '86. Editors: Tunna J.M. Engineering Materials Advisory Services Ltd, Warley 1986.



Rain-flow procedure I

Generally accepted solution today

Process decomposition: rainflow

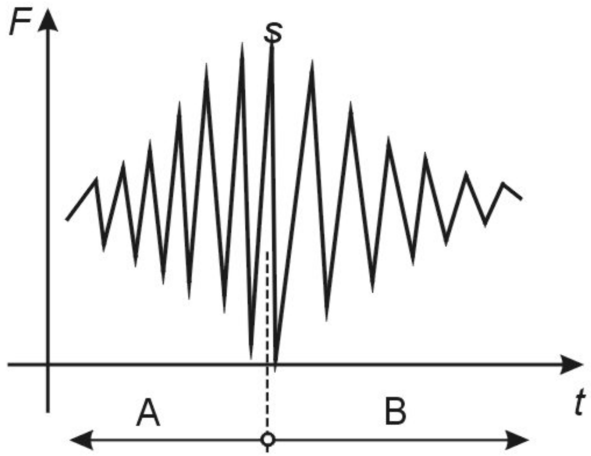


To znamená, že jsou nalezeny následující položky:

Horní<tab>Dolní<tab>Počet		
2	3	1.0
6	5	1.0
8	9	1.0
1	4	0.5
4	7	0.5
7	10	0.5

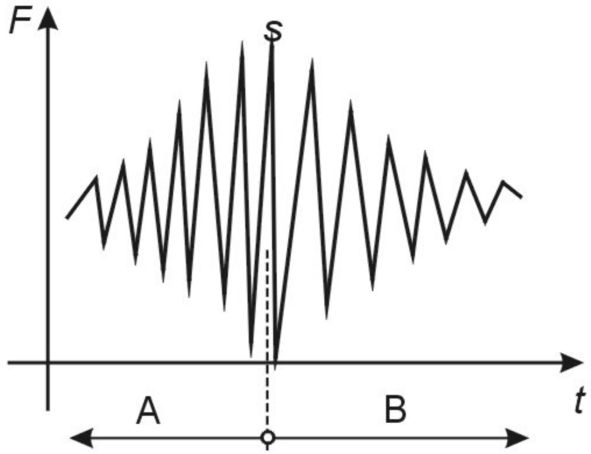
Residuum:

- Remnants after the first run – forming the cycle with largest amplitude



Rain-flow procedure II

- **How to process residuum?**
  1. **Keep unclosed half-cycle (their weight has to be set)**
  2. **Close in a second run (conservative)**



- **Threshold value**
  - **Every detected cycle is then analyzed by fatigue calculation – setting an adequate threshold for amplitude can shorten the processing time**

Free applications:

A. Nieslony: http://www.mathworks.com/matlabcentral/fileexchange/3026-rainflow-counting-algorithm

J. Papuga: http://www.pragtic.com - Program PragTic, sekce Tools->Loads->Decompose To Cycles

Rain-flow output

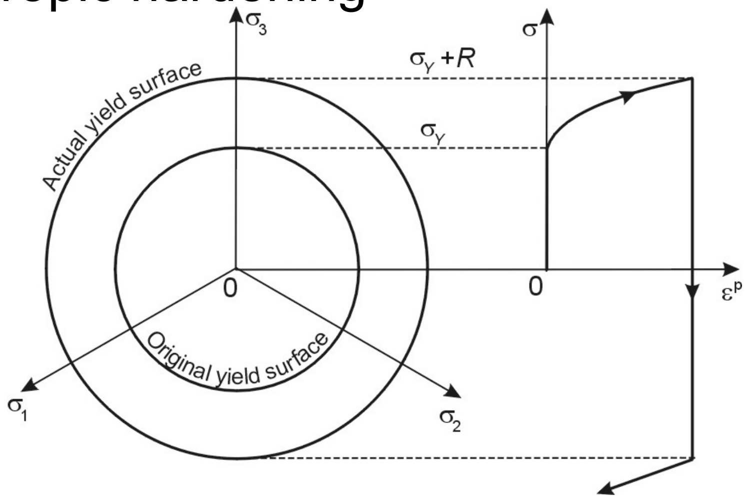
1. **Integrated with the fatigue analysis -> damage**
2. **Individual separated cycles (incl. the weight)**
3. **Rain-flow matrix (correlation matrix)**

M\A	6,25	18,75	31,25	43,75	56,25	68,75	81,25	93,75	106,25	118,75	131,25	143,75	156,25	168,75	181,25	193,75	206,25	218,75	231,25	243,75
-137,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-112,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-87,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-62,5	0	0	0	0	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-37,5	0	0	3	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-12,5	6	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12,5	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62,5	0	0	0	0	1	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0
87,5	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
112,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
137,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
162,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
187,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
212,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
237,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
262,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
287,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
312,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
337,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

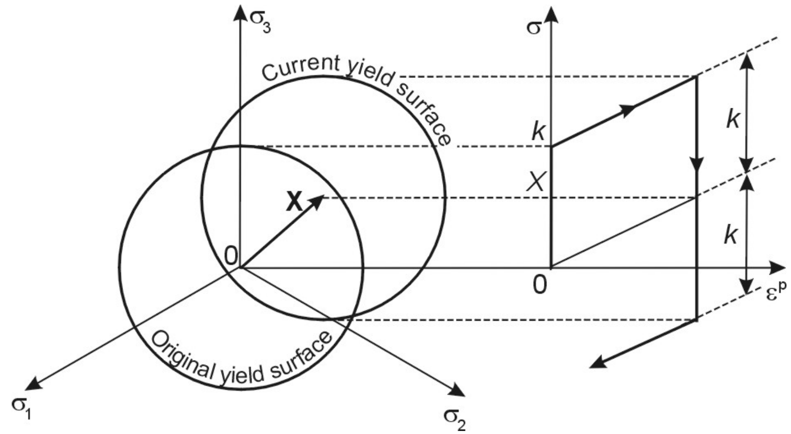
Rain-flow matrices

- **Useful to record very long load histories**
- **The order of cycles in the original load history is lost**
  - **Suitable:**
    - **High-cycle fatigue – more or less elastic loading, affected by small-scale kinematic hardening**

Isotropic hardening



Kinematic hardening

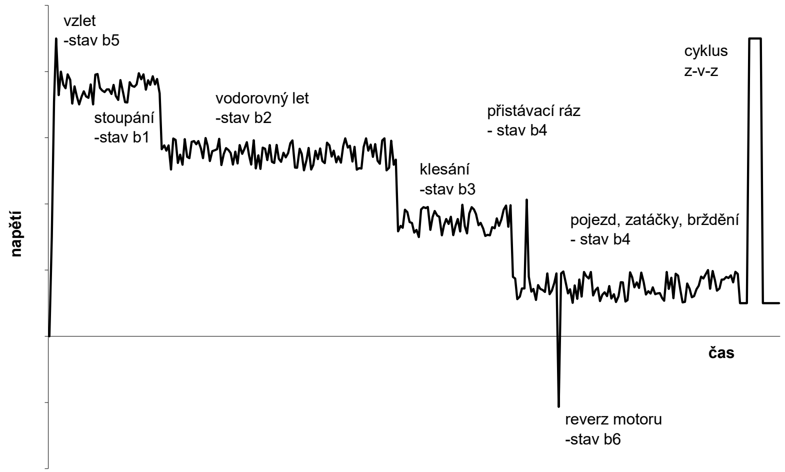


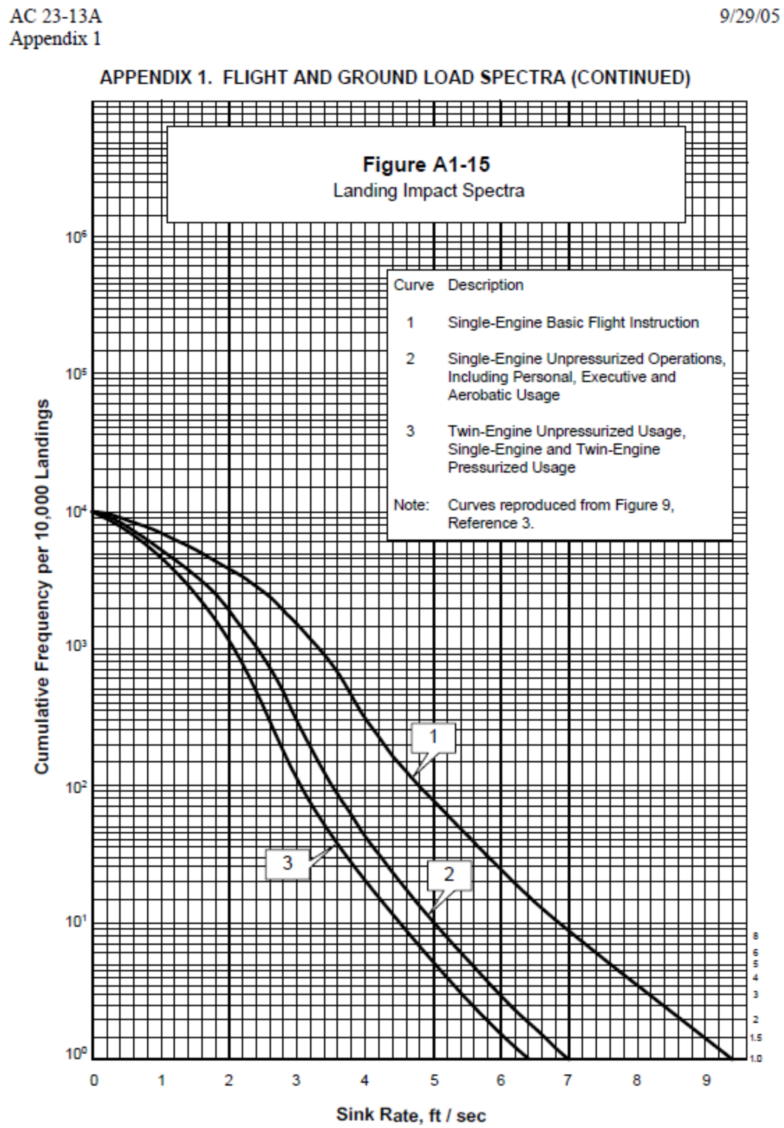
Load spectrum

Summary of decomposed load cycles related to a particular in-service operation

Aircrafts:

- Gust spectrum
- Taxi spectrum
- Aerobatic spectrum
- Landing spectrum
- Ground-Air-Ground





Damage accumulation

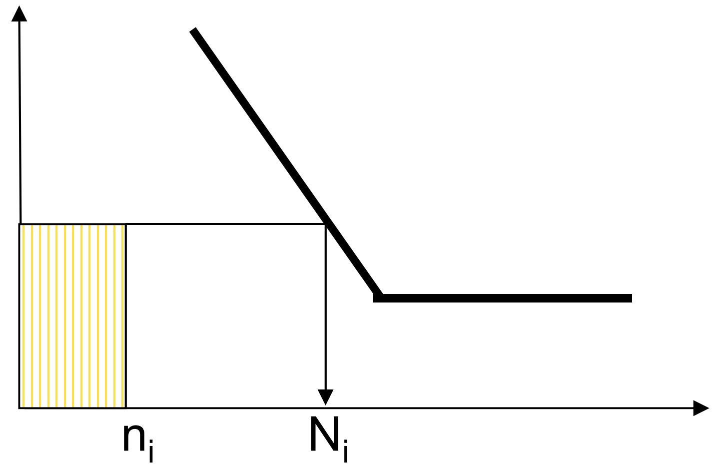
Number of cycles at a given load level

Fatigue damage

$$D_i = \left(\frac{n_i}{N_i}\right)$$

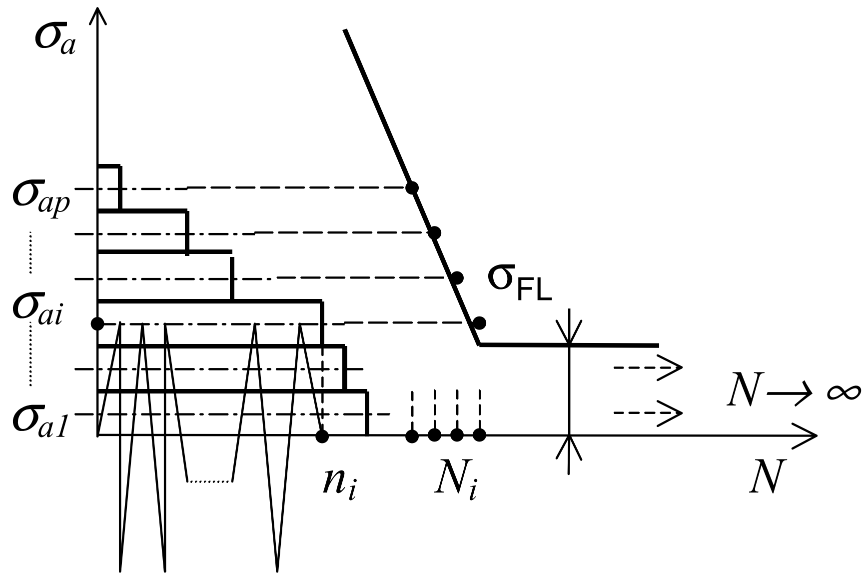
$$D_i = \left(\frac{n_i}{N_i}\right)^m$$

Number of cycles till break
at a given load level



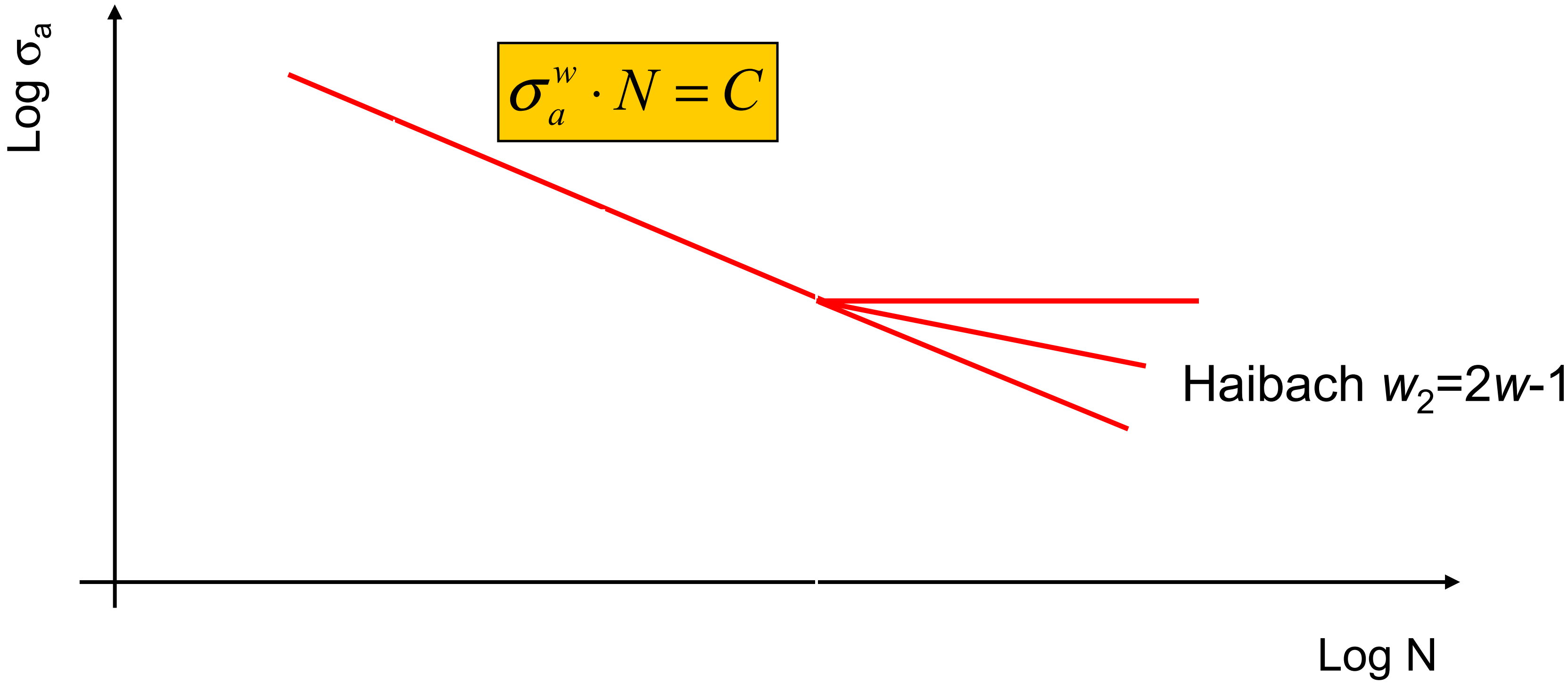
Linear damage accumulation
Palmgren, Miner 1945

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_p}{N_p} = \sum_{i=1}^{p} \frac{n_i}{N_i}$$



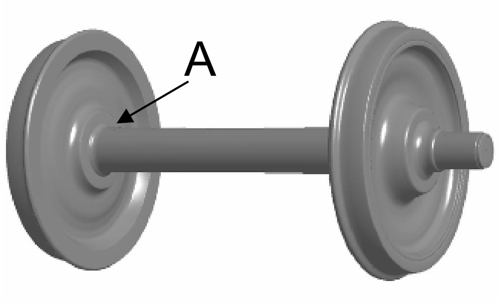
Below Fatigue Limit?

- **Loading with much bigger number of cycles**
- **Different concept can be used, see below**



Example – Fatigue life prediction

Estimate the damage of a railway axle caused by the provided load spectrum.



Input data: $S_{ult} = 780\,[\mathrm{MPa}]$

S-N curve of the axle steel

$$\sigma_a = \sigma'_f \cdot (2N)^b$$

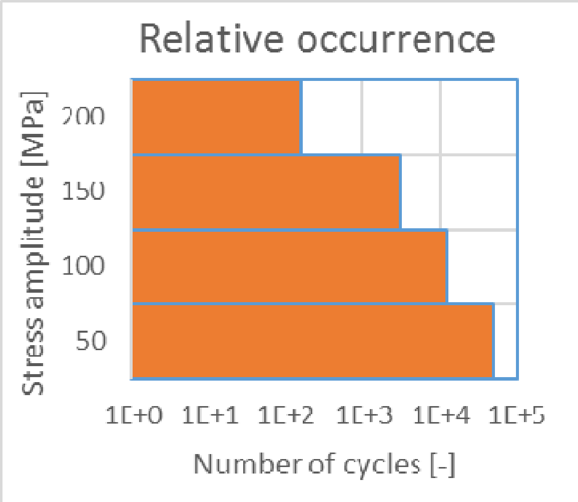
$$\sigma_{FL} = 1195 \cdot (2 \cdot 10^7)^{-0.077} = 327.5\ \mathrm{MPa}$$

Fatigue limit in the notch

$$\sigma_{FL,N} = 83.9\ \mathrm{MPa}$$

Histogram of nominal stresses (for 5000 km)

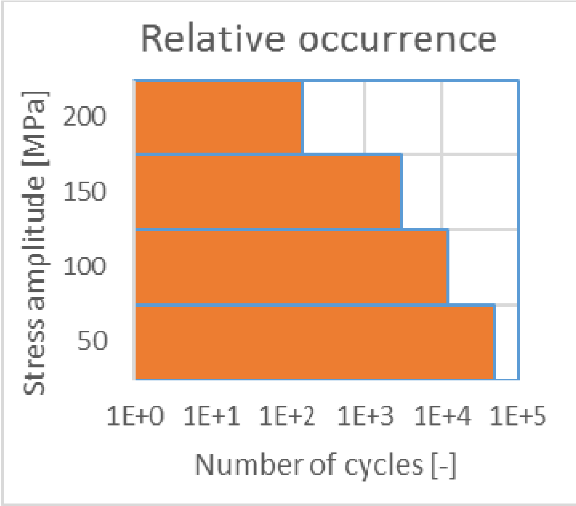
i	sig_ai [MPa]	n_i [-]
1	50	50000
2	100	12000
3	150	3000
4	200	150



Example – Continuation

Solution

i	sig_ai [MPa]	n_i [-]
1	50	50000
2	100	12000
3	150	3000
4	200	150

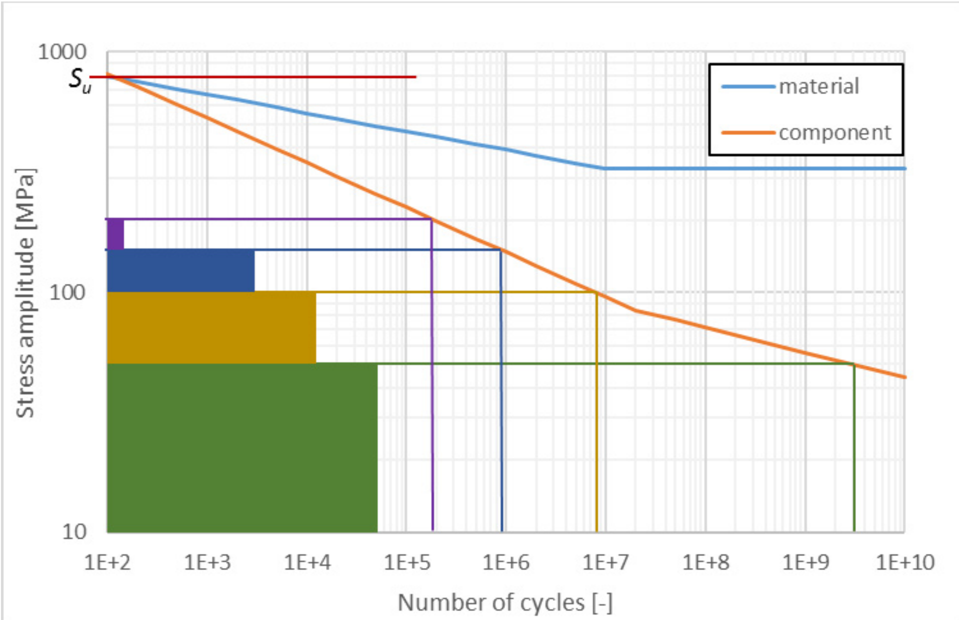


1. Estimation of the notch fatigue limit (at $2*10^7$ cycles):

$$\sigma_{FL,N} = \frac{327.5 \cdot 1.00 \cdot 0.67 \cdot 0.70 \cdot 1.00}{1.83} = 83.9 \text{ MPa}$$

2. Tensile strength -> derive the number of cycles N_{QS} at it
3. Basquin curve joins:
   $[N_{QS}; S_u]$ and $[N_{FL}; \sigma_{FL,N}]$
4. Below FL:
   Haibach w2=2w-1
   i.e.: b2=b/(2+b)
5. Partial damages and their sum:

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_p}{N_p} = \sum_{i=1}^{p} \frac{n_i}{N_i}$$



Example – Continuation

Fatigue damage

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_p}{N_p} = \sum_{i=1}^{p} \frac{n_i}{N_i}$$

i	sig_a	n_i	N_i	D_i
1	50	50000	3 080 570 453	1.62E-05
2	100	12000	7 797 390	0.001539
3	150	3000	885 264	0.003389
4	200	150	189 091	0.000793
			D_sum	0.005737

Number of the spectrum repeats to failure is

$$Z = \frac{D_{cr}}{D} = \frac{1}{D} = \frac{1}{0.05737} = 174.3$$

$$L = L_{50} = Z \cdot l = 174.3 \cdot 5000 = 871491 \text{ km}$$

But this is the mean fatigue life
(probability of failure 50%)