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# **Dynamická únosnost a životnost**

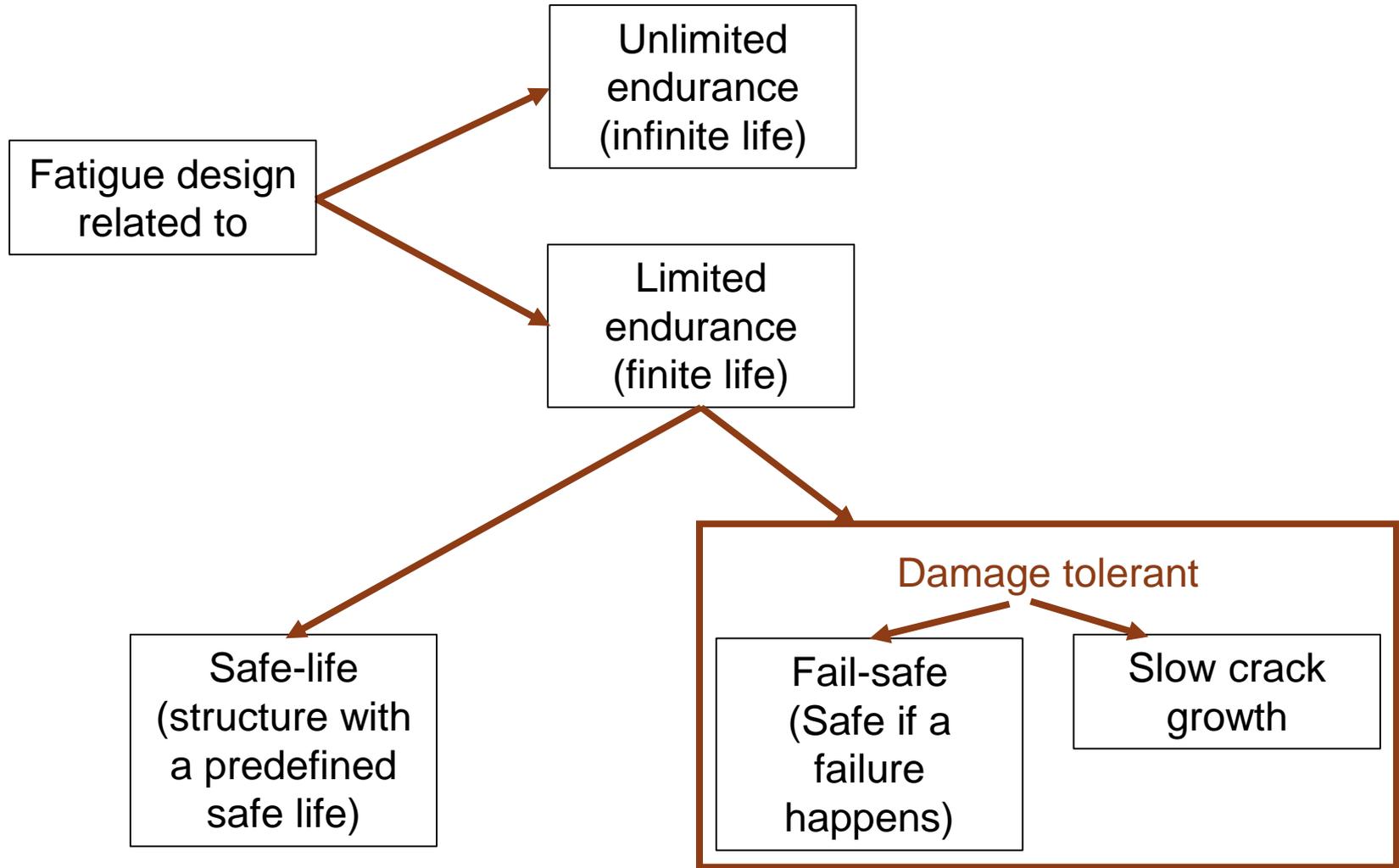
**Lekce 2**

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**Stress-Based Fatigue Analysis, Part #1**

**Jan Papuga**

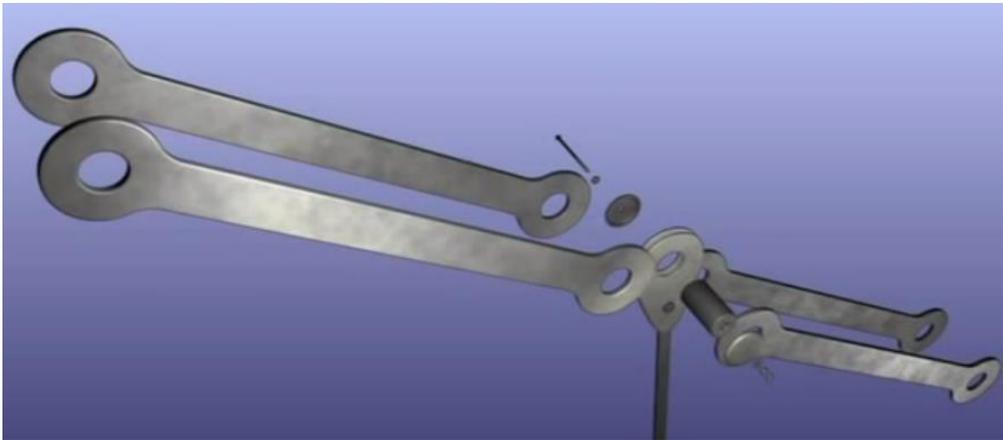
# Different Attitudes to Fatigue Design



# Silver Bridge Collapse - 1967



- Bridge over Ohio river collapsed in less than 1 minute. 46 casualties
- Was not fail-safe
- Corrosion crack in one member lead to complete disintegration
- Right practice here



<https://www.youtube.com/watch?v=dGQfUWvP0II>

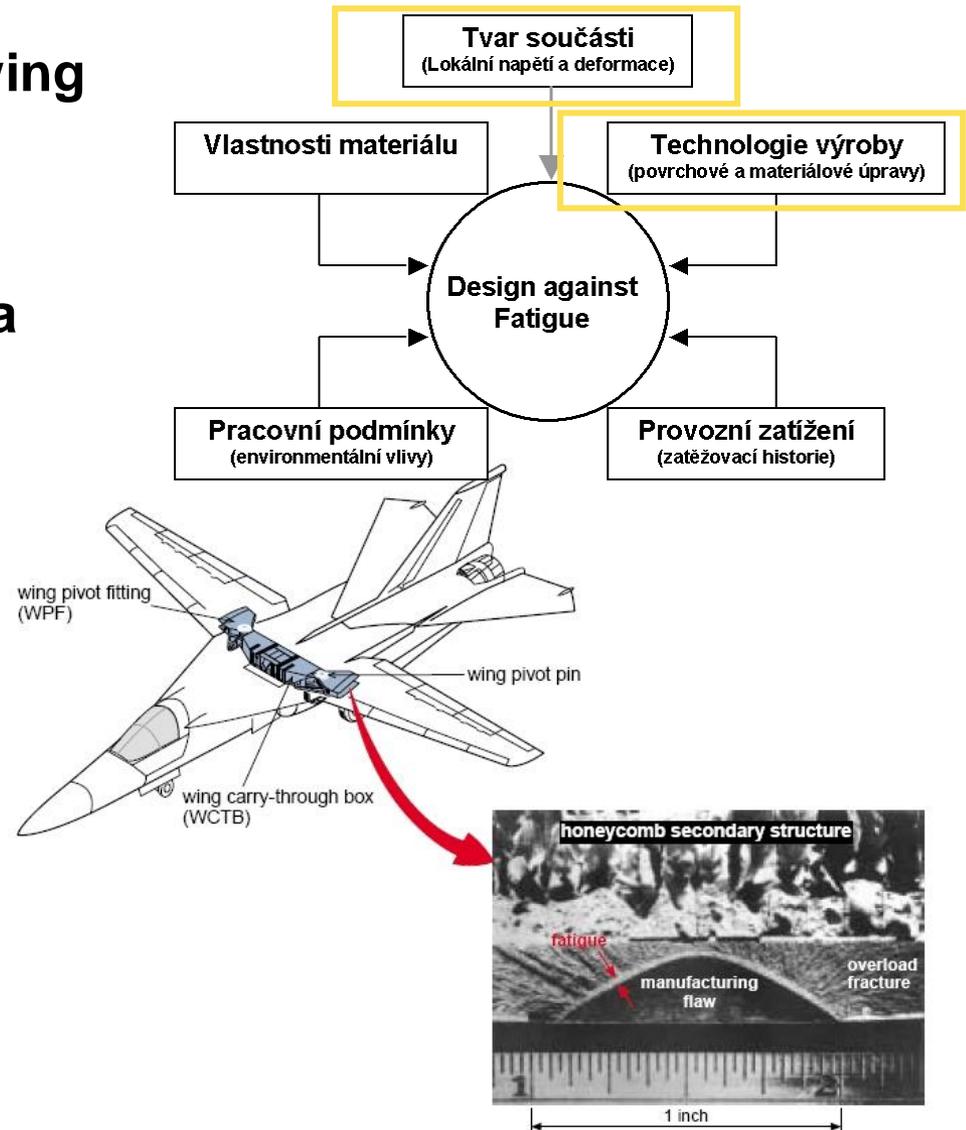
# General Dynamics F-111 (1969)

Bomber / fighter with variable wing configuration

F-111#94 lost a wing during a training flight (just one year a service)

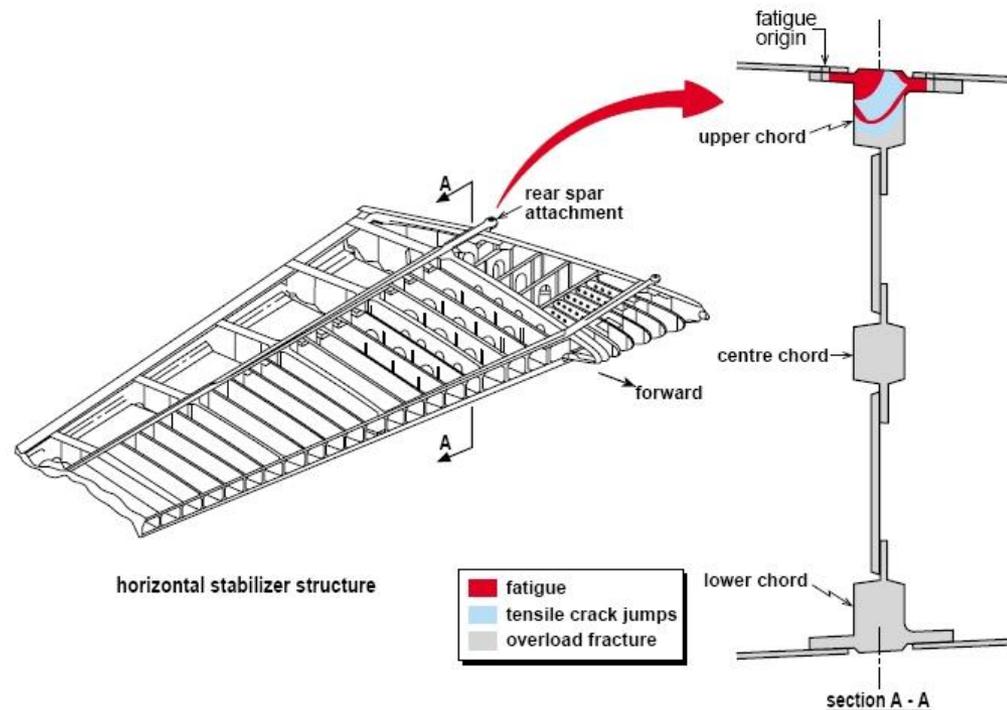
Reason:

- Large crack in the hinge already from the manufacturing process (23.4x5.9mm!) – only short growth (low fracture toughness of the high-strength steel)



# DAN-Air Boeing 707-300 (1977)

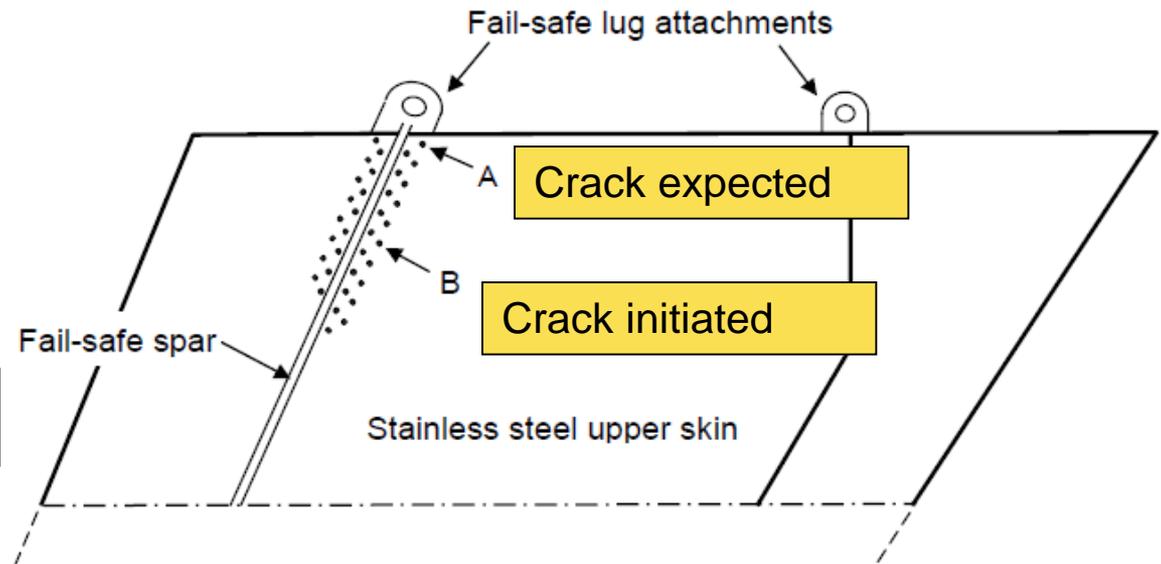
- Lost whole right stabilizer with elevator
- 47621 flight hours. 14 year old (60000 FH designed. 20 years)
- Reason: Stabilizer (its upper flange) designed as fail-safe but:
  - Skin material changed from 7075-T6 to steel. higher stiffness – was not supported by a full scale test
  - Only visual inspection prescribed
  - Once the flange broke. there was not enough time till the next inspection
  - Stabilizer was not fail-safe due to the chosen way of inspection



# DAN-Air Boeing 707 (1977)

## Consequences:

- **Fail-Safe is not guaranteed by design only. but also by the selection of the inspection method**
- **Material exchange can lead to stress redistribution and should be supported by**
  - **FEA**
  - **Experiment**



Schijve. J.: Fatigue Damage in Aircraft Structures. Not Wanted. But Tolerated?

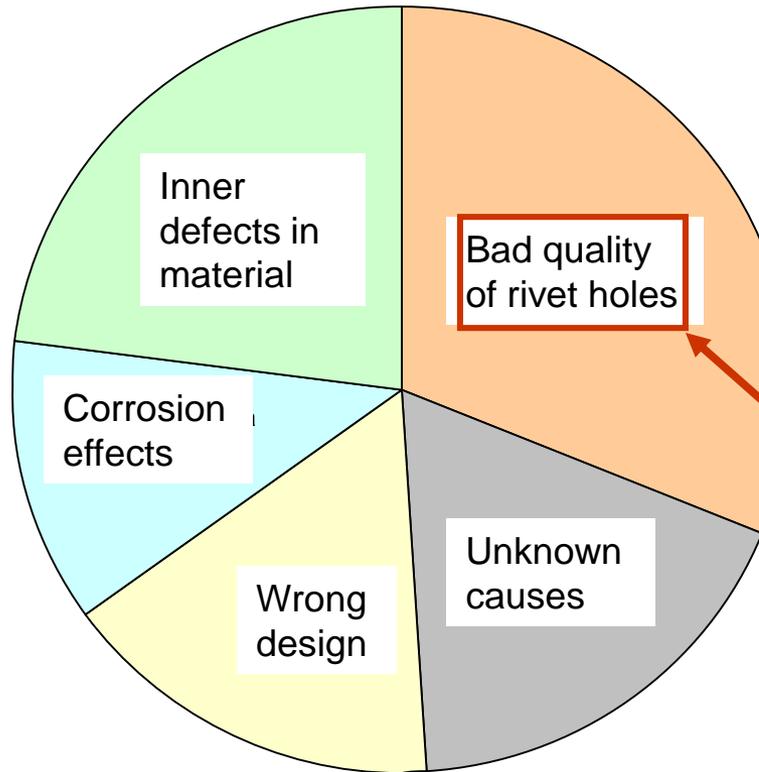
# Boeing 737 (1988)

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- Aircraft lost 5.5 m of the pressurized cabin
- 35496 FH. 89680 landings. 19 years in service. short flights. humid sea air
- Reason:
  - Quick joining of multiple small cracks in a row – Multiple Site Damage (MSD)



# Causes of fatigue issues



See NASA experiments on retired Boeing

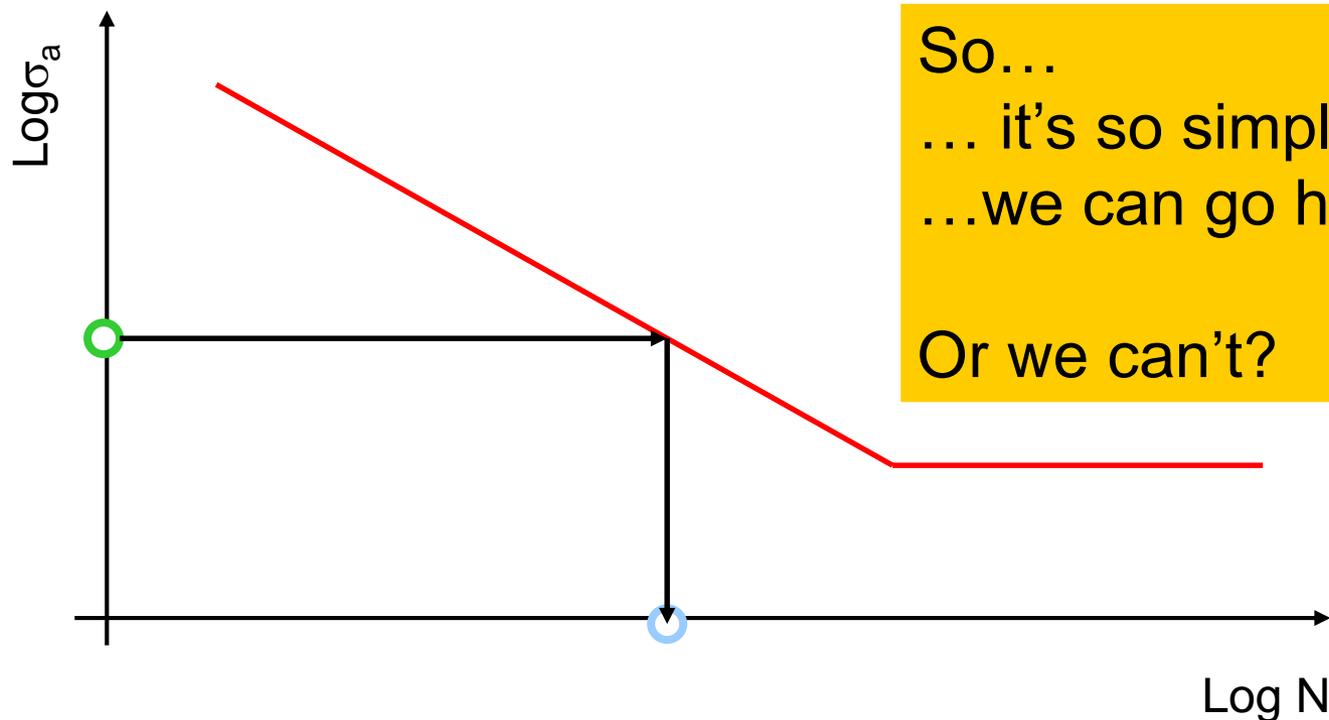
Sampath. S.G. and Simpson. D.: Airframe Inspection Reliability Under Field/Depot Conditions. Terms of Reference of AGARD Structures and Materials Panel Proposed Activity SC.77. October 1995.

# Intuitively

For inputs:

- Material with a known S-N curve
- Load amplitude  $\rightarrow$  FEA  $\rightarrow$  stress amplitude  $\sigma_a$

We can get immediately the final lifetime

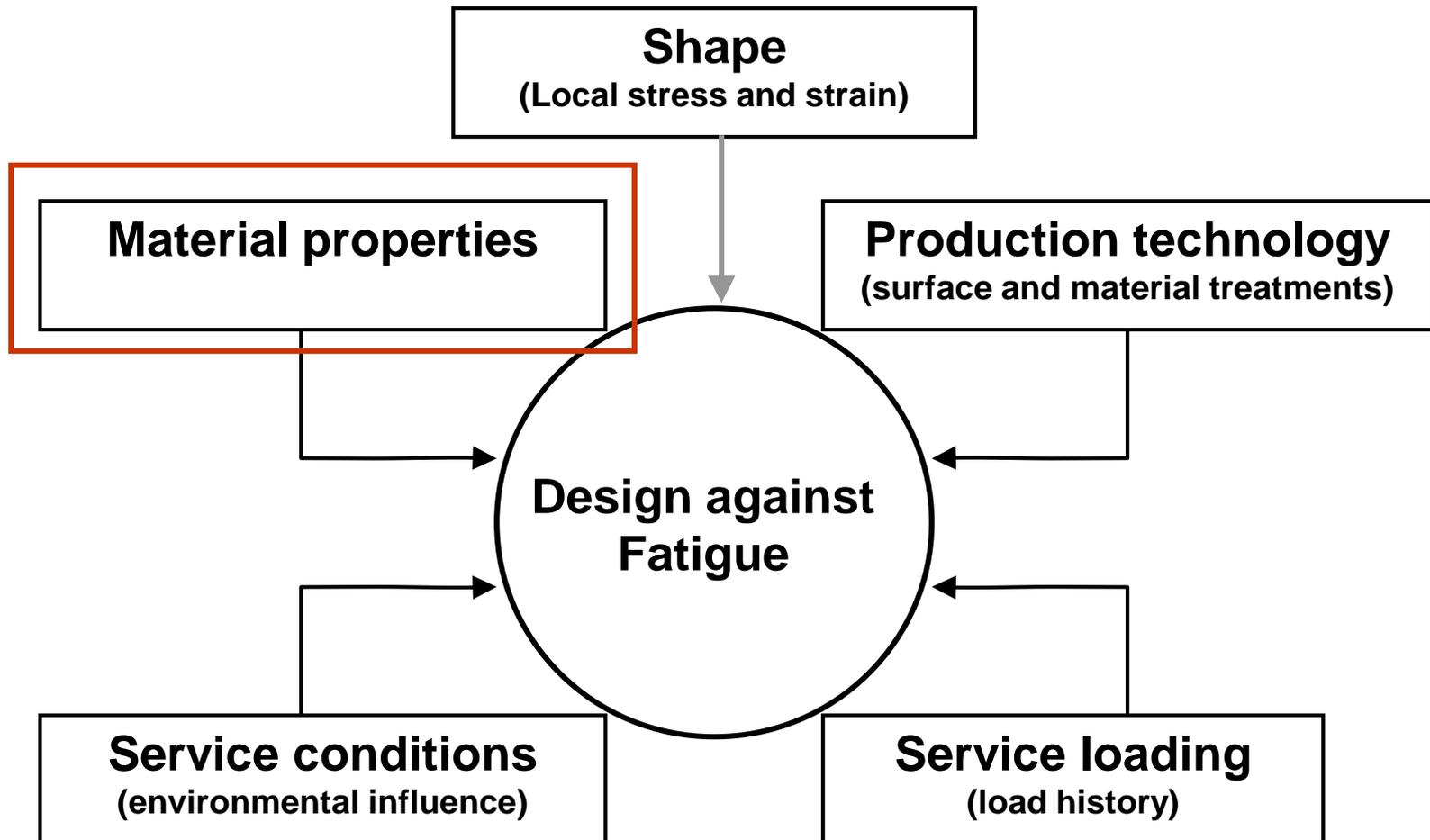


So...  
... it's so simple...  
...we can go home

Or we can't?

# Aspects influencing the fatigue life

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# Questions – Part I

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1. How fatigue design and static design of structures differ?
2. Typical attributes of low cycle fatigue and of high cycle fatigue?
3. Draw a hysteresis loop and describe on it elastic and plastic part of strain.
4. Specify phases of damage and fatigue progress in metals.
5. What is the main difference between safe-life and fail-safe design philosophy?
6. Which are main attributes of the damage tolerant design philosophy?
7. Define the fatigue limit of a material.
8. Which type of fatigue curve describes high-cycle fatigue primarily? Draw this curve.
9. Which type of fatigue curve describes low-cycle fatigue? Draw this curve.
10. Could be the fatigue limit higher than yield strength?
11. How can you estimate the fatigue limit of carbon steel from tensile strength?

# Questions – Part II

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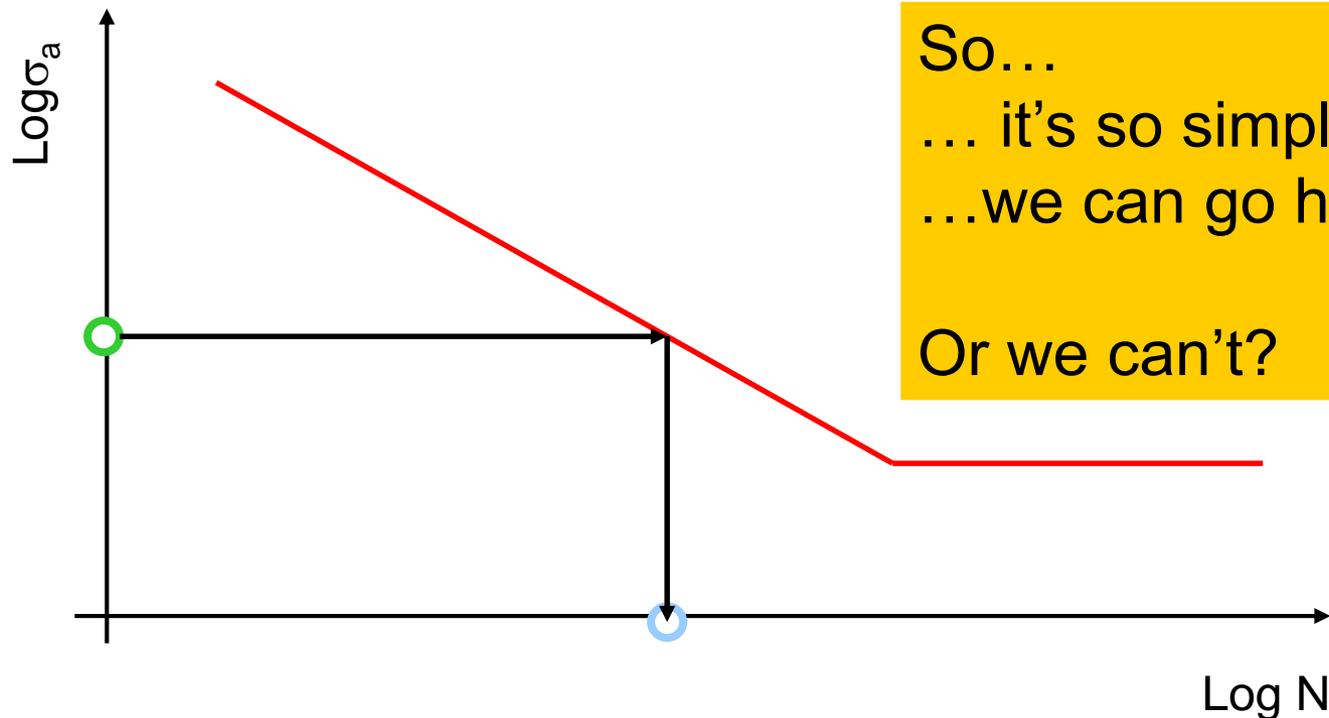
1. Example 1: Approximation of a stress amplitude is  $\sigma_a = K' \cdot \varepsilon_{ap}^{n'}$ . Derive equation for the total strain amplitude of a hysteresis loop  $\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} = ?$
2. Example 2: Approximation of the fatigue curve is  $\sigma_a^w \cdot N = C$  or  $\sigma_a = \sigma'_f (2N)^b$ . Derive relations between parameters  $C$ ,  $(\sigma'_f)$ ,  $b$ ,  $w$ .
3. Example 3: There are 6 material fatigue parameters  $K'$ ,  $n'$ ,  $\sigma'_f$ ,  $b$ ,  $\varepsilon'_f$ ,  $c$ , only are 4 independent. Derive relations between these parameters.
4. Example 4: There is special number of cycles ( $N_t$ ) in the Strain-life curve, where  $\varepsilon_{ae} = \varepsilon_{ap}$ . Derive equation to calculate this number  $N_t$ .

# Intuitively

For inputs:

- Material with a known S-N curve
- Load amplitude  $\rightarrow$  FEA  $\rightarrow$  stress amplitude  $\sigma_a$

We can get immediately the final lifetime



So...  
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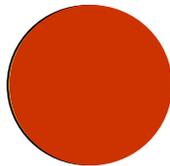
# Maximum stress area

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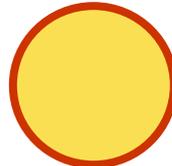
- **Fatigue ~ weak link mechanism**
- **If one link is damaged, the complete chain is broken**
- **The bigger area with a big stress, the bigger probability of damage**



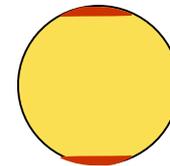
Fatigue limit: Push-pull < Rotating bending < Plane bending



Size effect



vs.



Stress distribution effect

# Fatigue limit modifying factors

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- Loading factor
- Size factor
- Surface quality factor
- Notch factor
- Mean stress effect
- Effect of thermo-mechanical treatment
- Temperature effect
- Multiaxiality factor
- Oxidation factor
- Hydrogen embrittlement factor
- Irradiation factor
- Load frequency factor
- ...

# Loading factor, $k_L$

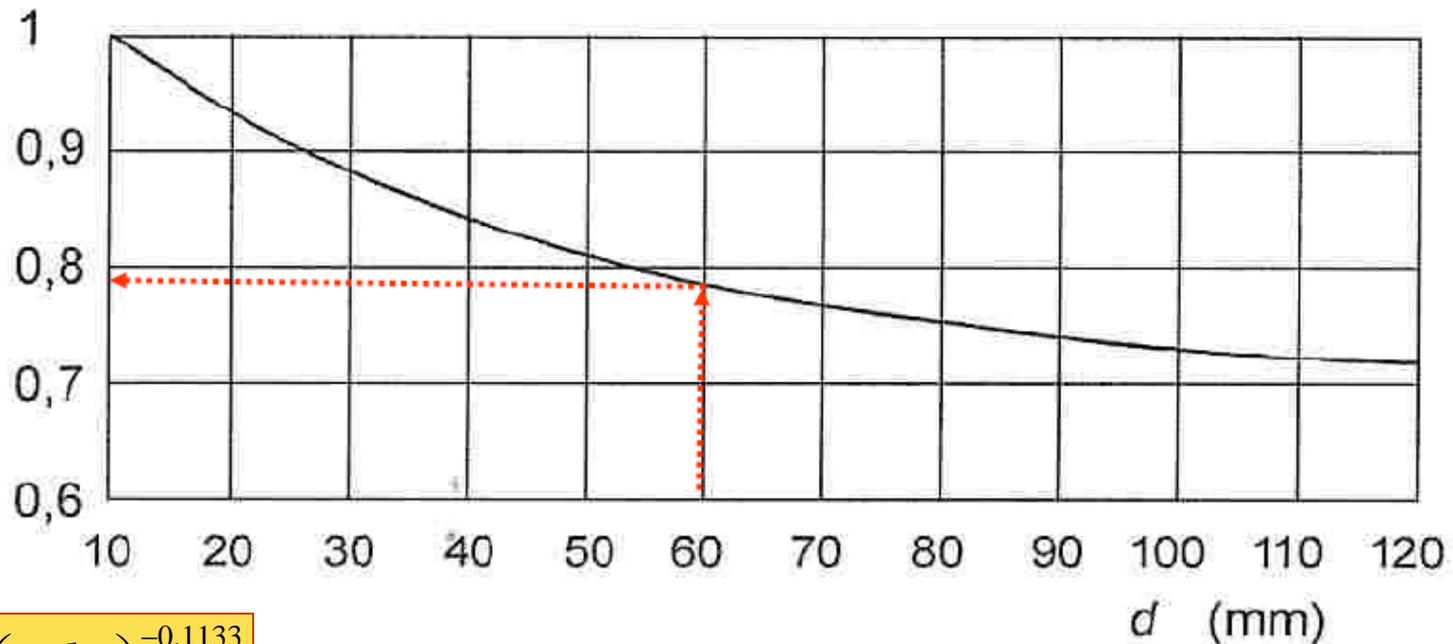
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- Historically, fatigue limits have been determined from simple bending tests with an intrinsic stress gradient in the test specimen.
- A specimen loaded in tension will have a lower fatigue limit than the one loaded in bending.
- An empirical correction factor, called the loading factor, is used to make an allowance for this effect.

<b><i>Loading Type</i></b>	<b><math>k_L</math></b>
Axial	0.9
Bending	1.0
Torsion	0.57

# Size effect

- Experimentally, larger parts have lower fatigue limits than smaller parts
- S-N curves are obtained from small specimens



$$k_s = \left( \frac{d}{7.62} \right)^{-0.1133}$$

0.3 inch

Set for  $d = 3 - 50 \text{ mm}$

Non-circular cross-section

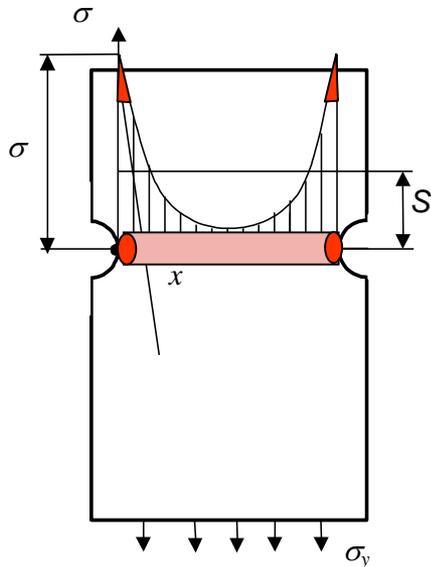
$$d = \sqrt{\frac{A_{0.95}}{0.077}}$$

# Size effect (exposed volume)

## Equivalent diameter

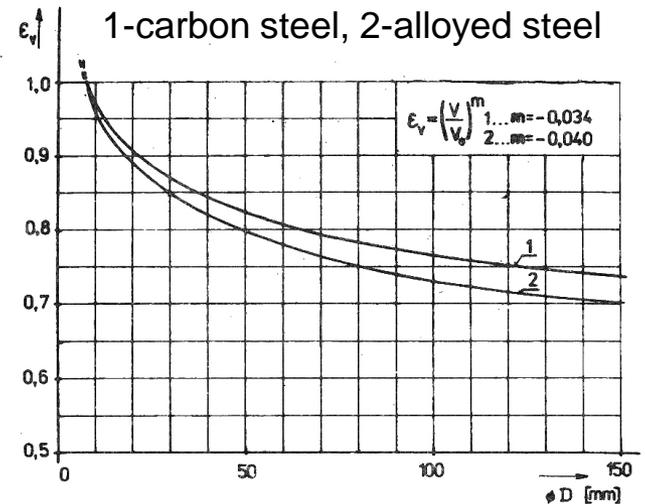
- the volume of the component loaded by stress exceeding **95%** of the maximum

$$k_S = \left( \frac{\sigma_C^D}{\sigma_C^{d=10}} \right) = \left( \frac{V_{\text{exp}}^D}{V_{\text{exp}}^d} \right)^m$$

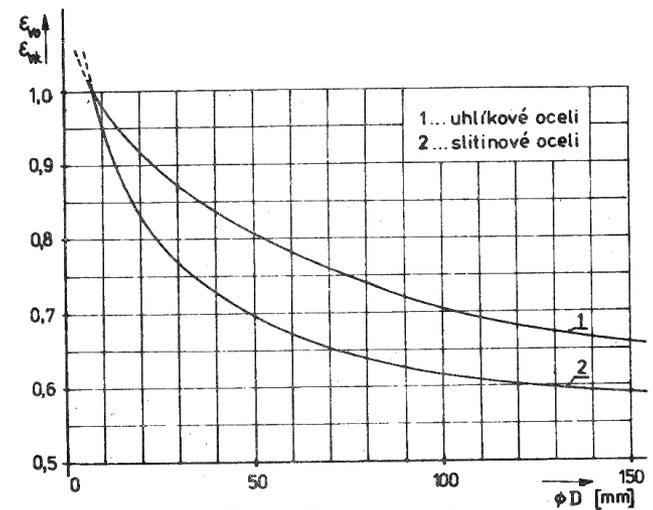


## Coefficient m

- Steels: -0,03 ÷ -0,06
- Structural steels: -0,034

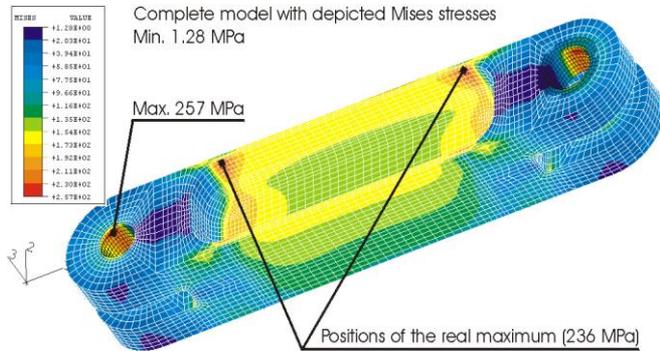


Push-pull



Bending, torsion

# Notch effect on fatigue properties

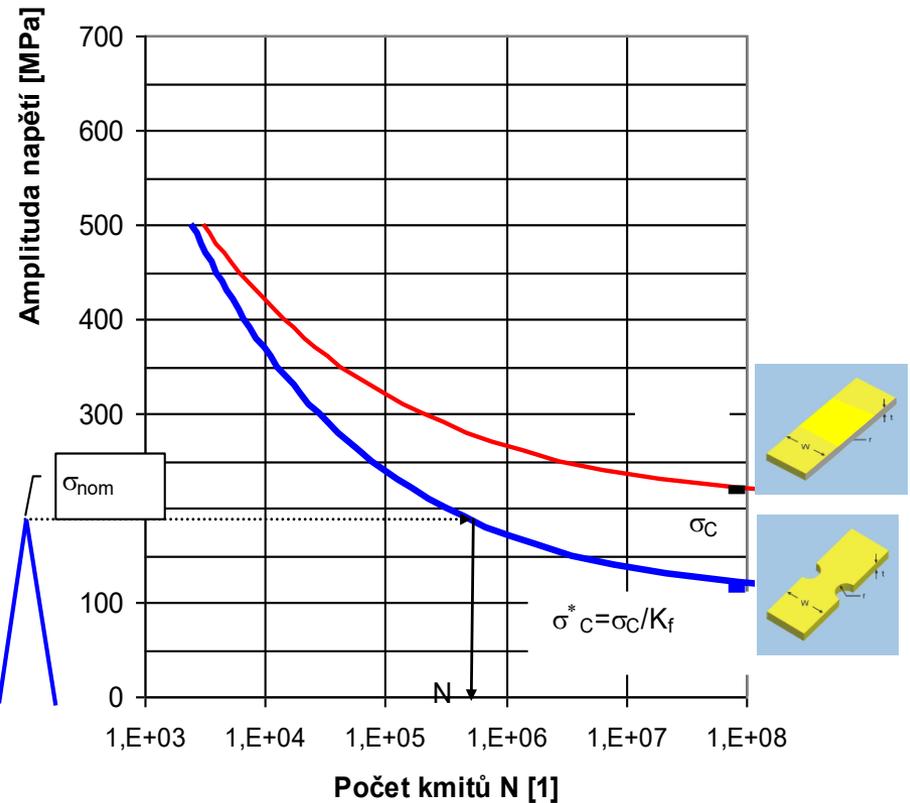


Stress concentration factor

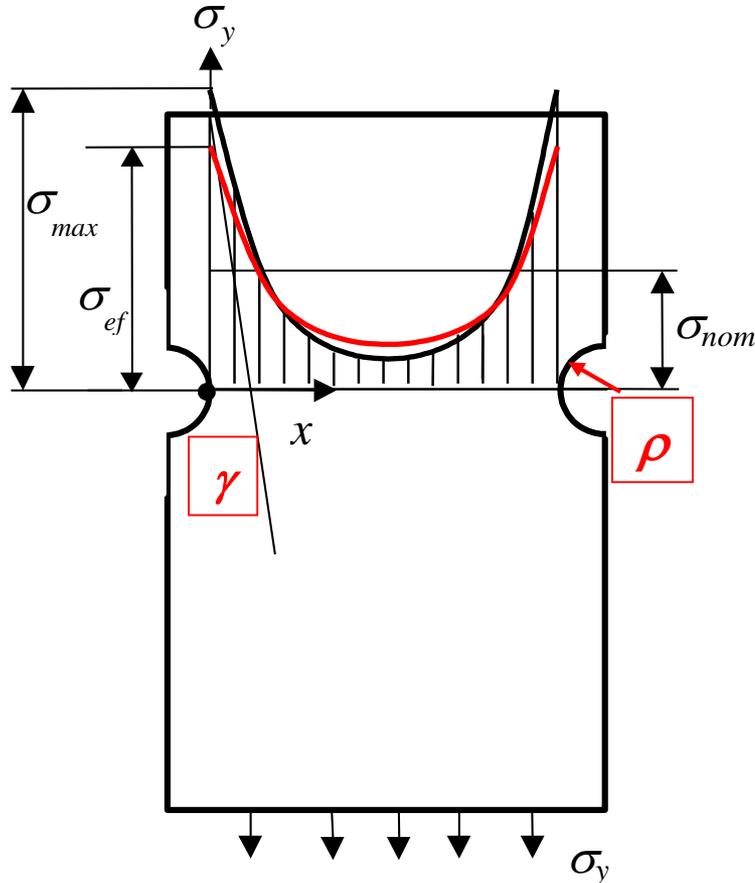
$$K_t \equiv \alpha = \frac{\sigma_{max}}{\sigma_{nom}}$$

Notch factor

$$K_f \equiv \beta = \frac{\sigma_C}{\sigma_C^*}$$



# Effect of notches – HCF



Relative stress gradient  $\gamma = \frac{1}{\sigma} \left( \frac{\partial \sigma_y}{\partial x} \right)_{x=0}$

Notch sensitivity effect

$$q = \frac{\sigma_{ef} - \sigma_{nom}}{\sigma_{max} - \sigma_{nom}} = \frac{\beta - 1}{\alpha - 1} \equiv \frac{K_f - 1}{K_t - 1}$$

Relation between both coefficients

$$K_f = 1 + (K_t - 1) \cdot q = \frac{K_t}{n}$$

Fatigue factor

$$n = \frac{K_t}{K_f}$$

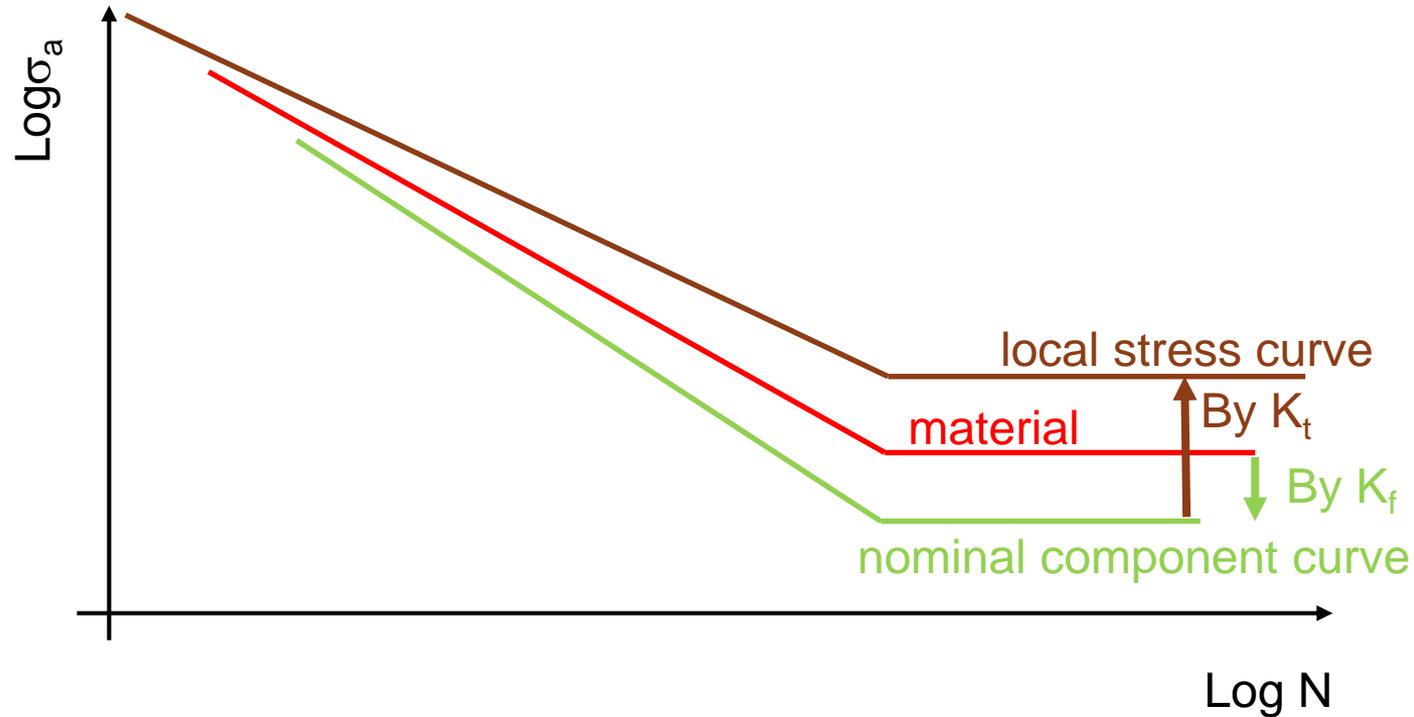
Two approaches:

$$n = f(\rho, R_m) \equiv n_\rho$$

$$n = f(\gamma, R_m) \equiv n_\gamma$$

# Notch factor modification

The S-N curve has to be modified to cover the transformation: **MATERIAL -> COMPONENT**



Fatigue limit:

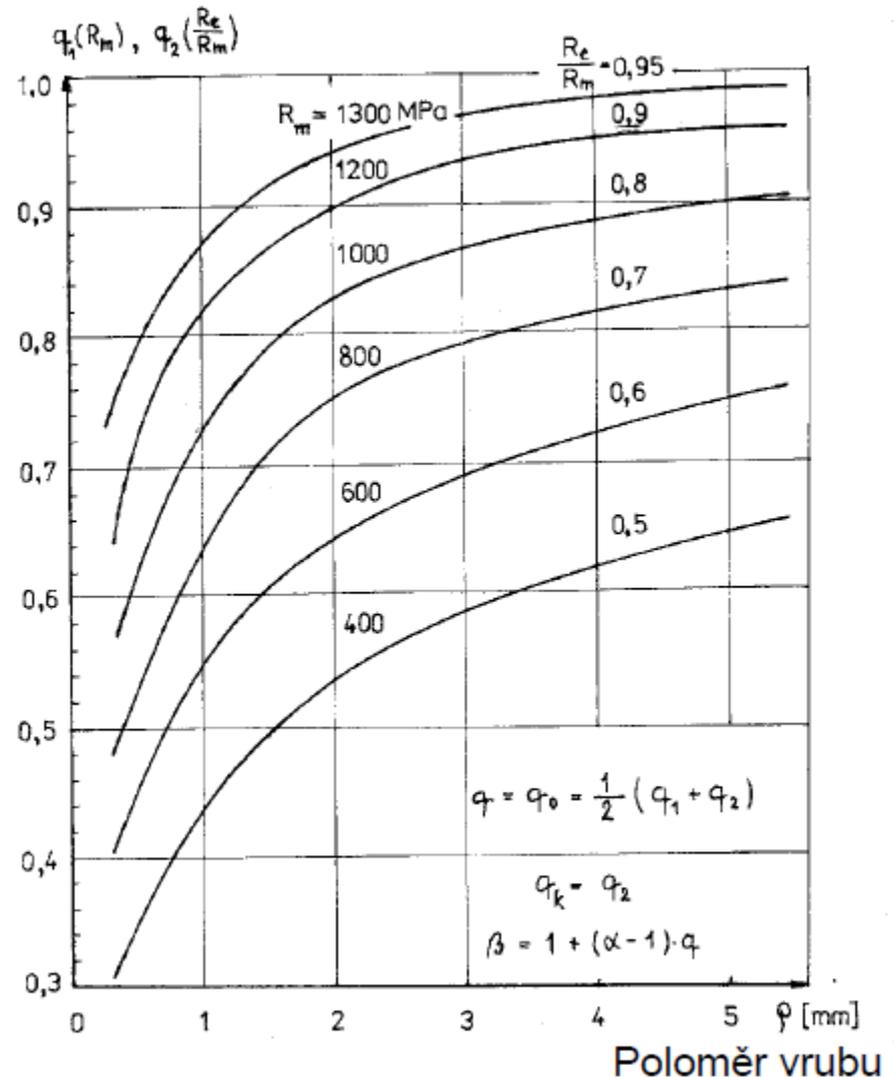
$\sigma_{FL}, \sigma_C$

$$K_f = \frac{\sigma_{FL,mat}}{\sigma_{FL,notched}} < K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

# Notch sensitivity $q$

$$K_f = 1 + (K_t - 1) \cdot q = \frac{K_t}{n}$$

- If  $q = 1$  then  $K_t = K_f$
- The more thermo-mechanically treated material, the higher notch sensitivity expectable
- It is not a material parameter



# Various Formulas to Include Notch Effect

$$n = \frac{K_t}{K_f}$$

**Notch factor**

**Fatigue factor**

$$n = f(\rho, R_m) \equiv n_\rho$$

**Thum**

$$K_f = 1 + (K_t - 1) \cdot q = \frac{K_t}{n}$$

$$n_\rho = \frac{K_t}{1 + (K_t - 1) \cdot q}$$

**Neuber**

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{A}{\rho}}}$$

$$n_\rho = \frac{K_t \left( 1 + \sqrt{\frac{A}{\rho}} \right)}{K_t + \sqrt{\frac{A}{\rho}}}$$

**Peterson**

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{a}{\rho}}$$

$$n_\rho = \frac{K_t \left( 1 + K_t \frac{a}{\rho} \right)}{K_t + \frac{a}{\rho}}$$

**Heywood**

$$K_f = \frac{K_t}{1 + 2 \frac{K_t - 1}{K_t} \sqrt{\frac{a'}{\rho}}}$$

$$n_\rho = 1 + 2 \frac{K_t - 1}{K_t} \sqrt{\frac{a'}{\rho}}$$

# If FEA Results Processed - Fatigue Factor

**Volejnik, Kogaev,  
Serensen**

$$n = f(\gamma, R_m) \equiv n_\gamma$$

$$n_\gamma = 1 + \left( \frac{1}{v_\infty} - 1 \right) \cdot \left( \frac{L/\gamma}{2\pi d_0 / \frac{2}{d_0}} \right)^{-\mu}$$

**Eichlseder (FEMFAT)**

$$n_\gamma = 1 + \left( \frac{\sigma_{-1,b}}{\sigma_{-1}} - 1 \right) \cdot \left( \frac{\gamma}{2/d_0} \right)^{K_D}$$

**Siebel, Stiller**

$$n_\gamma = 1 + \sqrt{c \cdot \gamma}$$

**FKM-Richtlinie**

$$\gamma' \leq 0,1 \text{mm}^{-1}$$

$$0,1 \text{mm}^{-1} < \gamma' \leq 1 \text{mm}^{-1}$$

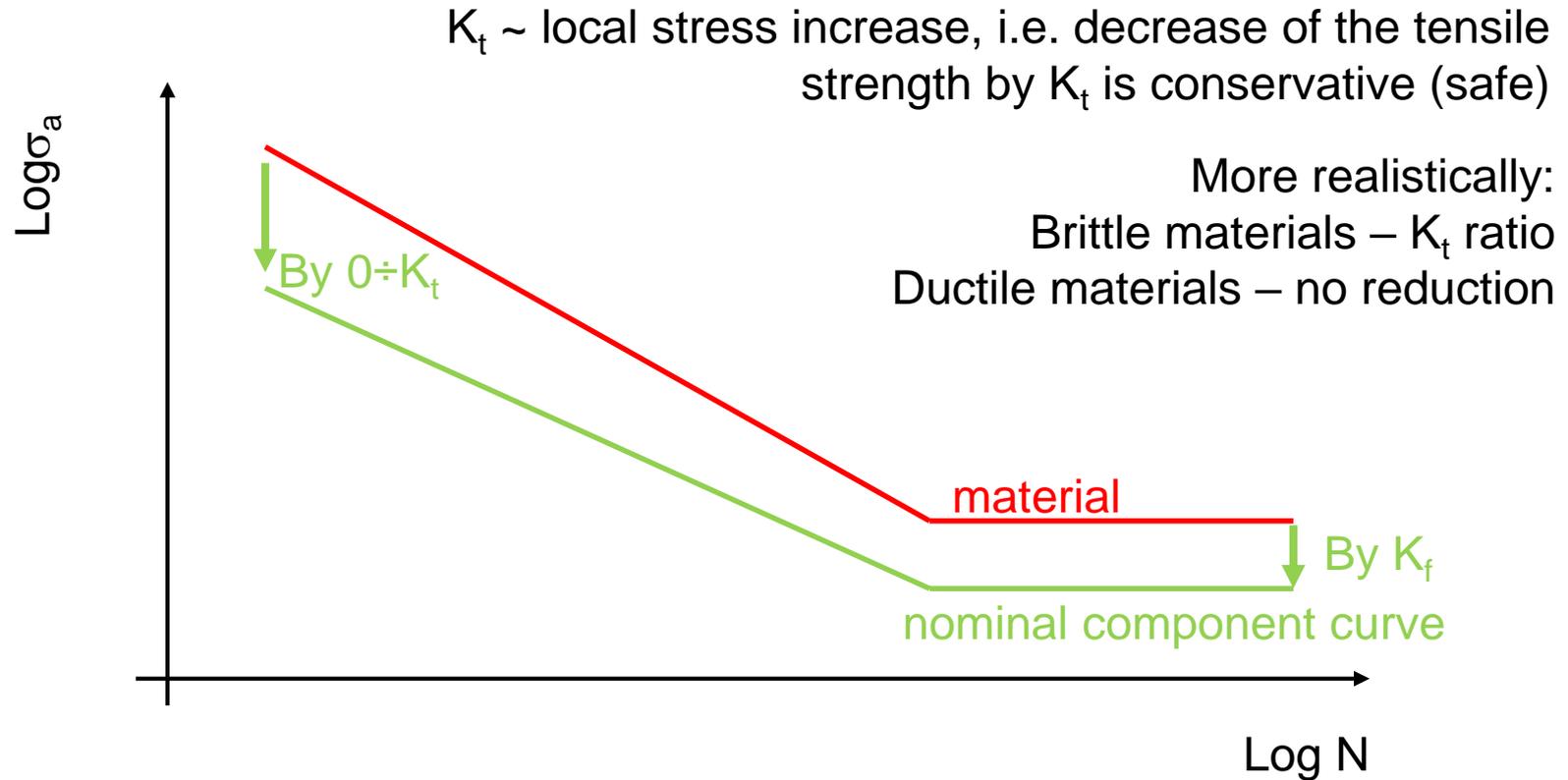
$$1 \text{mm}^{-1} < \gamma' \leq 100 \text{mm}^{-1}$$

$$n_\gamma = 1 + \gamma' \cdot 10^{-\left( a_G - 0,5 + \frac{R_m}{b_G} \right)}$$

$$n_\gamma = 1 + \sqrt{\gamma'} \cdot 10^{-\left( a_G + \frac{R_m}{b_G} \right)}$$

$$n_\gamma = 1 + \sqrt[4]{\gamma'} \cdot 10^{-\left( a_G + \frac{R_m}{b_G} \right)}$$

# Lower lifetimes?



Fatigue limit:

$\sigma_{FL}$ ,  $\sigma_C$

$$K_f = \frac{\sigma_{FL,mat}}{\sigma_{FL,notched}} < K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

**In addition to the nominal approach** (using  $K_f$ ), there are two basic concepts for local stress evaluation. **The basic premise:** Stress directly at the notch:

- 1) Relates to  $K_t$  factor
- 2) Should not be used directly without any modification for fatigue life estimate, which depends on  $K_f$
- 3) If used, the result is likely to be conservative

## **Stress gradient:**

- Its value helps to compute fatigue factor  $n$ , which shifts the **effective material curve upwards**

## **Critical distance:**

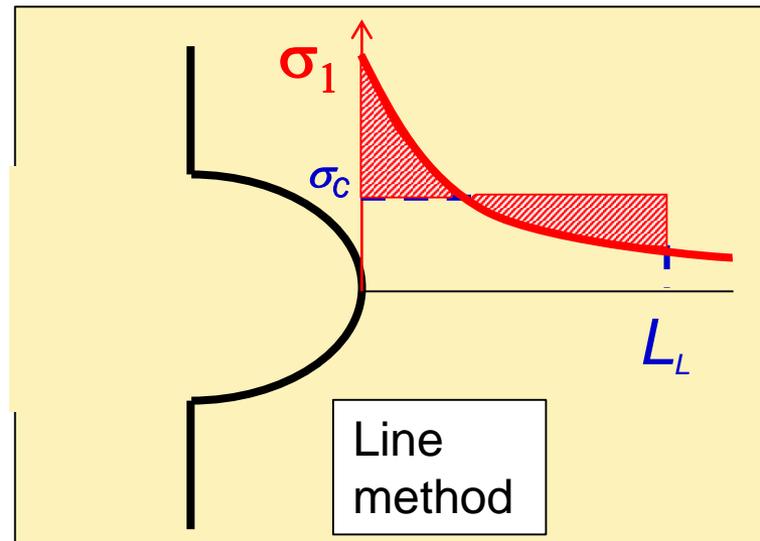
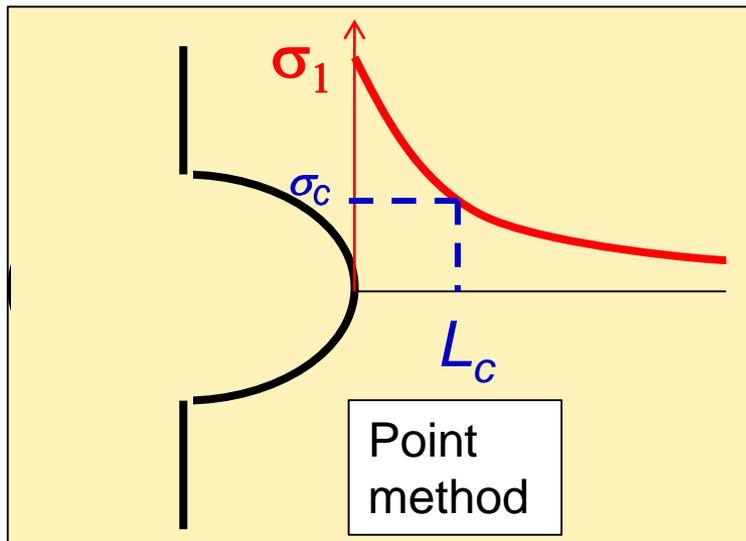
- The stress used for calculating the **fatigue life** is derived **in a particular distance** from the notch root
- **the effective stress is thus lower**, than it would be at the notch

# Theory of Critical Distance (TCD)

Simplifies the critical volume method to a 1D problem.

**Theorem: The crack initiation at the notch starts in the moment, when some reference value (Sig HMH, Sig1, Damage Parameter P) in a defined depth below the surface reaches critical value:**

- Point method (crit. depth  $L_c$ )
- Line method (analyses the integral mean of the stress from the notch root to the distance  $L_L$ )



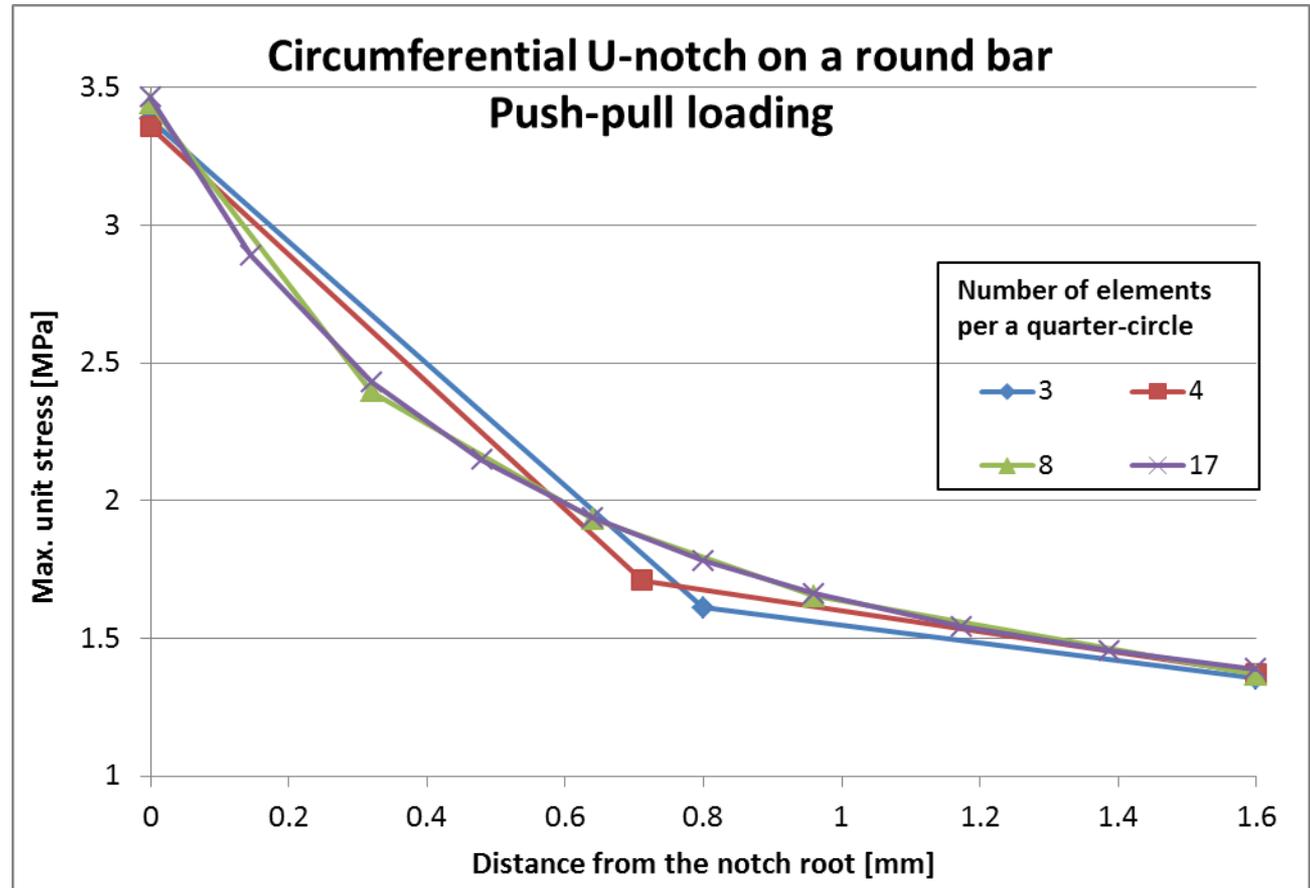
# Summary – Notch Effect

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- The notches decrease the fatigue life and fatigue strength
- Nominal solution works through notch sensitivity factor  $q$  and stress concentration factor  $K_t$  to estimate notch factor  $K_f$ , and to modify the S-N curve accordingly
- If directly the local stress value read from the FEA-results is used, the resulting life value is likely to be conservatively underestimated.
- This is the reason, why some correction has to happen:
  - TCD: Decrease the processed stress (and use original S-N curve)
  - Stress gradient: Modify the material curve (and use original local stress)
- Though such methods have been already implemented in some commercial fatigue solvers, the extent of validation has to be doubted.

# Effect of the FE-model quality

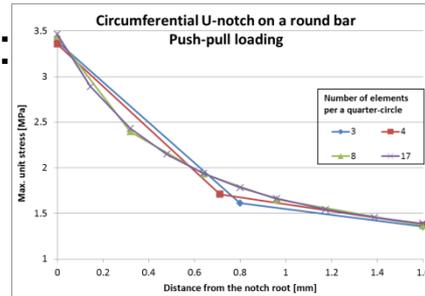
- Best results for equi-sided elements - bricks
- The sizes of element sides can differ for some variants here – this is the reason for only slight difference between variants with 3 and 4 elements
- Common mesh quality in static analyses of airplanes in Evektor – 2 elements per a quarter-circle



# Push-pull load case

- Relative stress gradient (RSG):

$$\gamma' = \frac{\gamma}{\sigma_{max}} = \frac{1}{\sigma_{max}} \frac{d\sigma}{dx}$$



## Note:

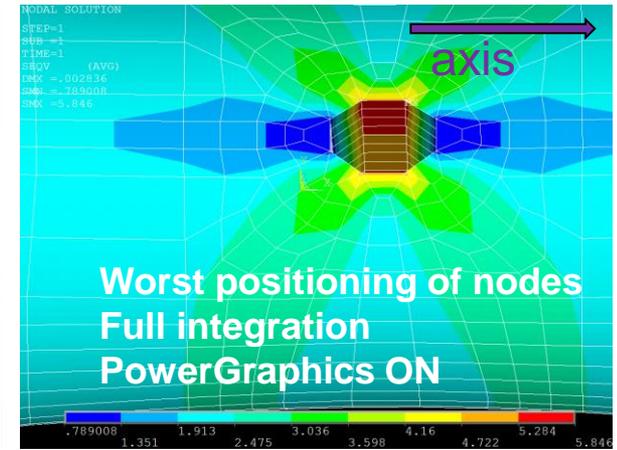
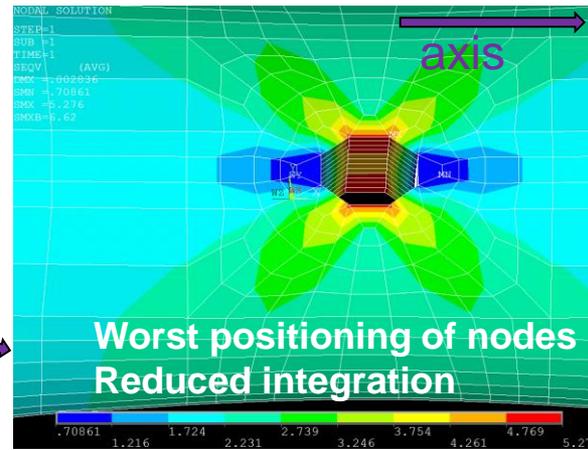
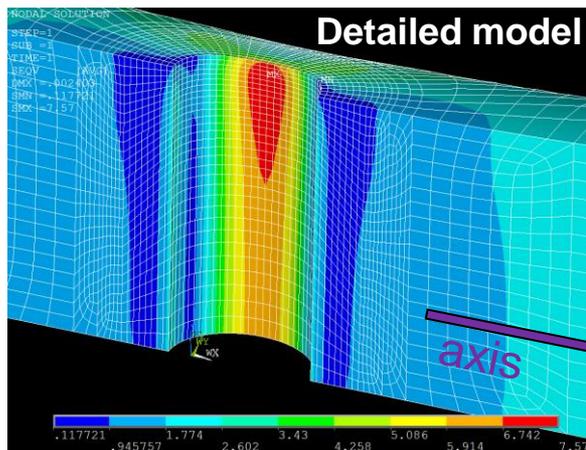
Only minor differences in results of maximum stress are likely to be caused by a simple load mode with the obvious hot-spot location, in which the node is positioned.

- If we compare various notches here:
  - RSG by common models are close to one half of the right value
  - Minimum effect of the mesh quality on the SCF (see  $K_{t,net}$ )
  - First principal stress results in less scattered analysis error compared with von Mises stress

Specimen type (radius of the notch root)	equivalent stress	parameter	ideal FE-model	Evektor FE-model	relative deviation from the ideal value
U-notch (R=1.6 mm), real model uses 3 elements per a quarter-circle	von Mises stress	r.s.g. [1/mm]	1.610	0.836	-48%
		Kt, net [-]	2.133	2.076	-3%
	1st principal stress	r.s.g. [1/mm]	1.203	0.655	-46%
		Kt, net [-]	2.360	2.306	-2%
Fillet (R=0.4 mm), real model uses 3 elements per a quarter-circle	von Mises stress	r.s.g. [1/mm]	5.953	2.821	-53%
		Kt, net [-]	2.469	2.229	-10%
	1st principal stress	r.s.g. [1/mm]	4.618	2.452	-47%
		Kt, net [-]	2.717	2.543	-6%
V-notch (R=1.6 mm), real model uses 3 elements per a quarter-circle	von Mises stress	r.s.g. [1/mm]	1.582	0.778	-51%
		Kt, net [-]	2.242	2.177	-3%
	1st principal stress	r.s.g. [1/mm]	1.141	0.603	-47%
		Kt, net [-]	2.506	2.453	-2%
Hole (R=2.0 mm), real model uses 2 elements per a quarter-circle	von Mises stress	r.s.g. [1/mm]	1.056	0.486	-54%
		Kt, net [-]	2.513	2.476	-1%
	1st principal stress	r.s.g. [1/mm]	0.814	0.420	-48%
		Kt, net [-]	2.628	2.609	-1%

# Push-pull load case

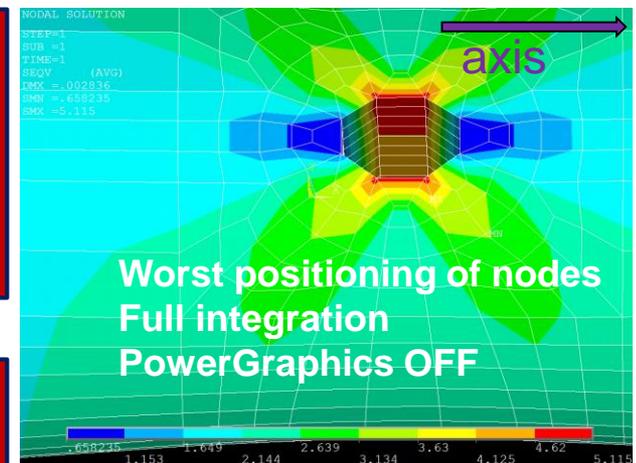
**Beware:** Small maximum stress errors are likely to be caused by simple load modes and a priori known critical location. If a node is not placed to the maximum stress location, the output can be much worse.



**Output:** At unknown critical place, stress can be underestimated by

$$\text{Relative error} = \frac{7.57 - 5.115}{5.115} = 48.0\%$$

while neglecting other errors (wrong critical place location, much smaller stress gradient).



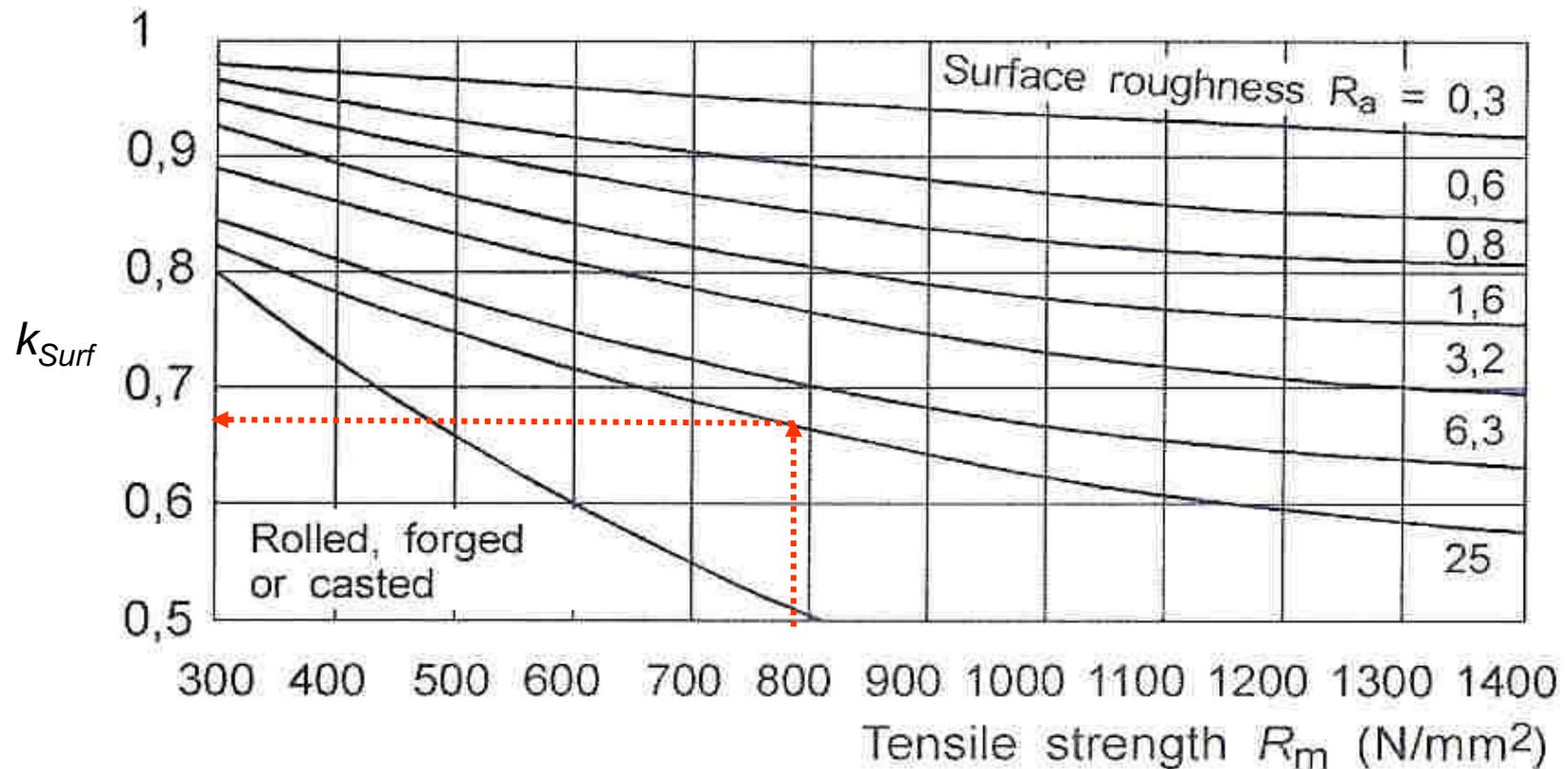
All problems with the setup of postprocessing are **caused by rough mesh**, and should not occur for better meshes.

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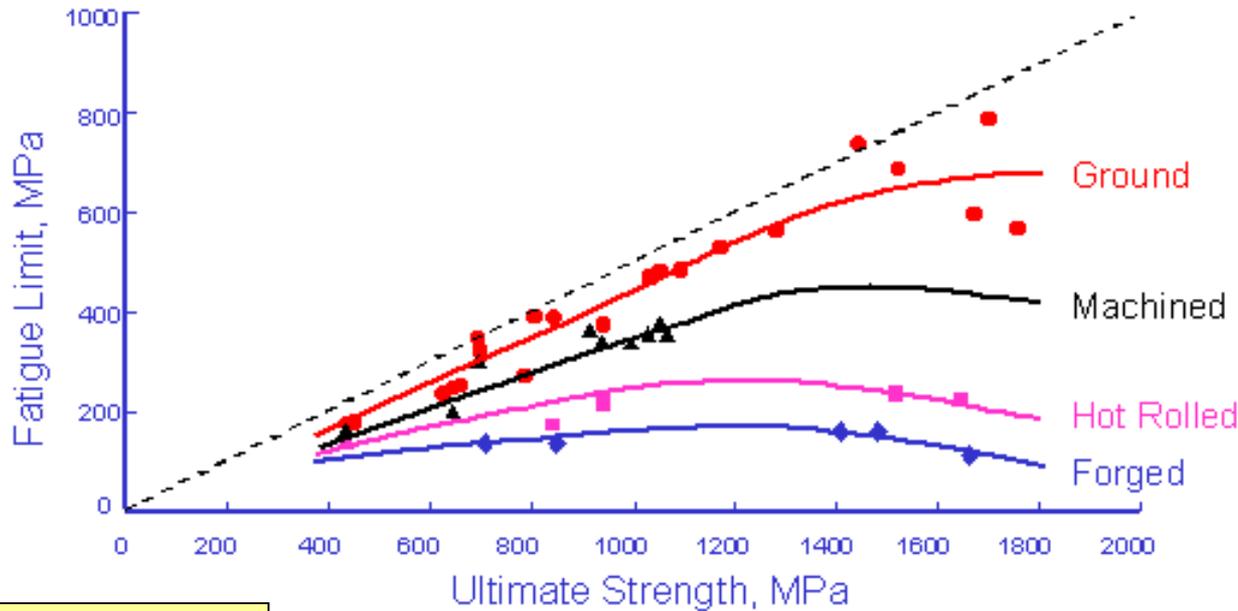
# Other fatigue limit modifying factors

# Fatigue surface quality

$$k_{Surf} = \frac{\sigma_c^{real}}{\sigma_c^{specimen}}$$



# Surface Roughness Effect



Noll and Lipson:  
Allowable Working  
Stresses. Society  
for Experimental  
Stress Analysis,  
Vol. III, no. 2,  
1949

$$k_{Surf} = a \cdot R_m^\beta$$

	$a$	$\beta$
Ground	1.58	-0.085
Machined	4.51	-0.265
Hot rolled	57.7	-0.718
Forged	272	-0.995

**These are mean regression  
curves, not the safe  
ones!**

Source: [www.efatigue.com](http://www.efatigue.com)

# Surface Roughness Effect – Part II

## FKM-Richtlinie

$$k_{Surf} = 1 - a \cdot \log R_z \cdot \log \left( \frac{2 \cdot R_m}{R_{m,N,min}} \right)$$

Prepared for analyzing normal stresses, if shear stresses concerned, the factor has to be multiplied by a parameter  $f_{W,t}$

$R_z$  Mean roughness in microns

$R_m$  Tensile strength in MPa

	Steel	Cast steel	Cast iron with spheroidal graphite	Tempered cast iron	Grey cast iron	Wrought aluminum alloys	Cast aluminum alloys
	Ocel	Litá ocel	Litina s kuličkovým grafitem	Temperovaná litina	Šedá litina	Hliníkové slitiny tvářené	Hliníkové slitiny lité
$a$	0.22	0.2	0.16	0.12	0.06	0.22	0.2
$R_{m,N,min}$	400	400	400	350	100	133	133
$f_{W,t}$	0.577	0.577	0.65	0.75	1	0.577	0.75

# Effect of Thermo-Mechanical Treatment - $k_T$

$$k_{Tec} = \frac{\sigma_c^{real}}{\sigma_c^{specimen}}$$

FKM-Guideline: Analytical Strength Assessment of Components in Mechanical Engineering. 5th revised edition. Frankfurt/Main, Forschungskuratorium Maschinenbau (FKM) 2003.

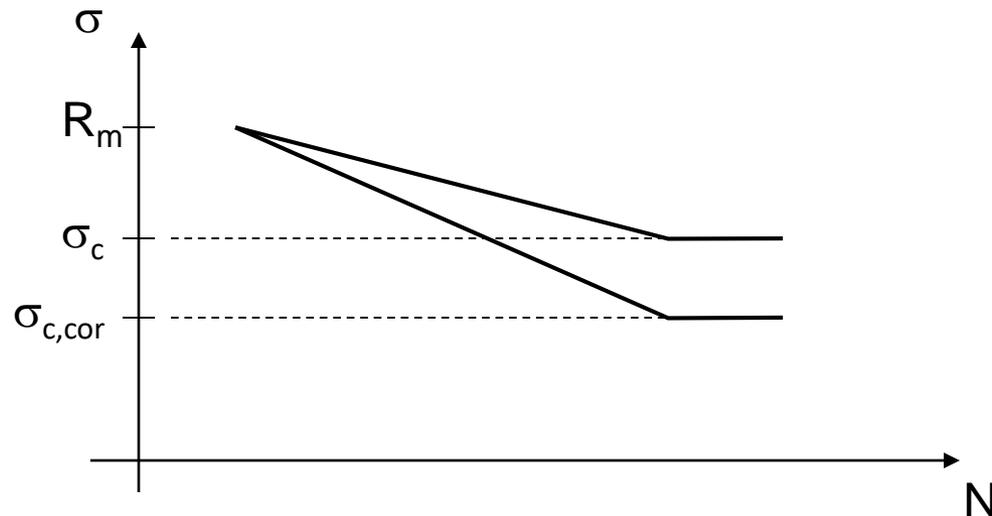
	unnotched	notched
<b>Steel</b>		
<b>Chemo-thermal treatments</b>		
<b>Nitriding</b> Depth of case 0,1...0,4 mm Surface hardness 700 to 1000 HV 10	1,10 - 1,15 (1,15 - 1,25)	1,30 - 2,00 (1,90 - 3,00)
<b>Case hardening</b> Depth of case 0,2 ... 0,8 mm Surface hardness 670 to 750 HV 10	1,10 - 1,50 (1,20 - 2,00)	1,20 - 2,00 (1,50 - 2,50)
<b>Carbo-nitriding</b> Depth of case 0,2 ... 0,8 mm Surface hardness 670 to 750 HV 10	(1,80)	
<b>Mechanical treatment</b>		
<b>Cold rolling</b>	1,10 - 1,25 (1,20 - 1,40)	1,30 - 1,80 (1,50 - 2,20)
<b>Shot peening</b>	1,10 - 1,20 (1,10 - 1,30)	1,10 - 1,50 (1,40 - 2,50)
<b>Thermal treatment</b>		
<b>Inductive hardening</b> <b>Flame-hardening</b> Depth of case 0,9 ... 1,5 mm Surface hardness 51 to 64 HRC	1,20 - 1,50 (1,30 - 1,60)	1,50 - 2,50 (1,60 - 2,80)

# Summary: Effects of Surface State

The most influencing for high number of cycles

Thermo-mechanical processing of the surface layer affects its properties

- Mild changes in static properties
- Pronounced effect around fatigue limit
- i.e. the effect increases with increasing the desired fatigue life



# Fatigue Limit of a Notched Part

$$\sigma_C^{\text{real}} = \frac{\sigma_C^{\text{smooth specimen in push-pull}}}{K_{f,tot}}$$

**Němec, Puchner, Linhart**

$$K_{f,tot} = \frac{K_f}{k_S \cdot k_{Surf} \cdot k_{Tec}}$$

**Volejnik, Kogaev, Serensen**

$$K_{f,tot} = \left( \frac{K_f}{k_S} + \frac{1}{k_{Surf}} - 1 \right) \cdot \frac{1}{k_{Tec}}$$

**FKM Guideline**

$$K_{f,tot} = \left( K_f + \frac{1}{k_{Surf}} - 1 \right) \cdot \frac{1}{k_S \cdot k_{Tec}}$$

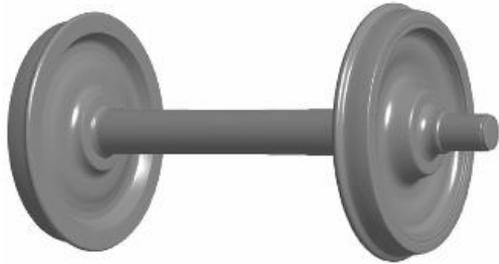
**Eichlseder**

$$K_{f,tot} = \frac{k_{statistic}}{k_S \cdot k_{Tec} \cdot k_{temp} \sqrt{k_{grad} - 1 + k_{Surf}^2}}$$

# Fatigue limit of a real part

---

Train wheel set:



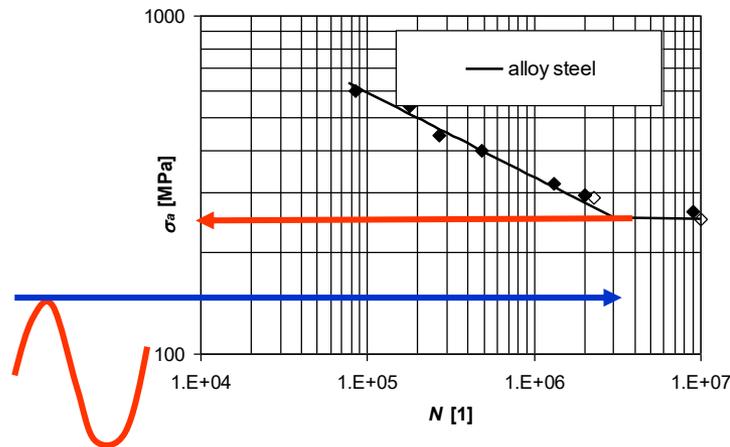
Target life?

$$\sigma_{FL,N} = \frac{\sigma_{FL} \cdot k_L \cdot k_{SF} \cdot k_S \cdot k_T}{K_f}$$

# Safety factor for unlimited fatigue life

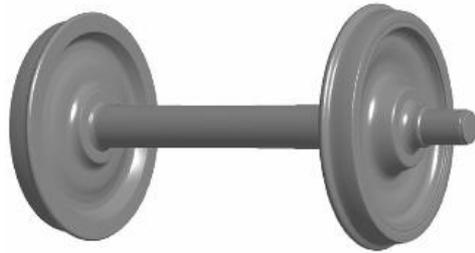
## 1. Alternating stress ( $R=-1$ )

- In-service loading stress amplitude  $\sigma_a$
- fatigue limit of the real part in the critical cross section area  $\sigma_{FL,N}$



$$n_\sigma = \frac{\sigma_{FL,N}}{\sigma_a}$$

# Example – Fatigue safe factor calculation



## Problem description:

### Railway axle

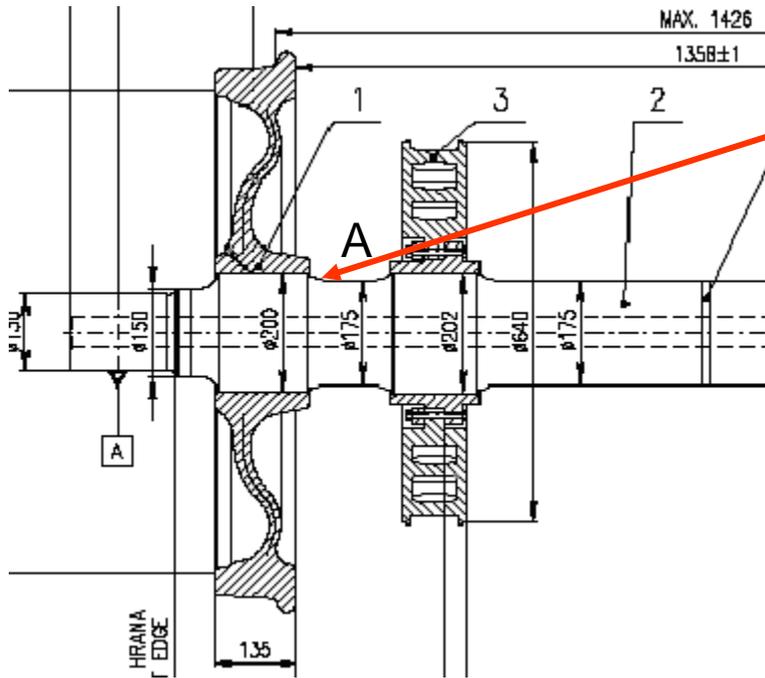
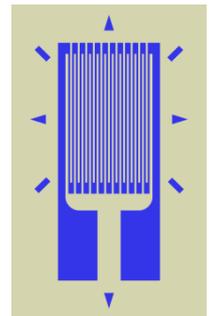
Material: alloy steel 25CrMo4,  
ASTM 4130

Point A of the potential crack  
initiation

Experimentally measured strain  
amplitude (in the point A):

$$\varepsilon_{a,\max} = 312 \text{ [microstrain]}$$

Strain gauge (measures  
resistance changes):



# Example – continuation

<http://www.efatigue.com>

## Material Property Finder

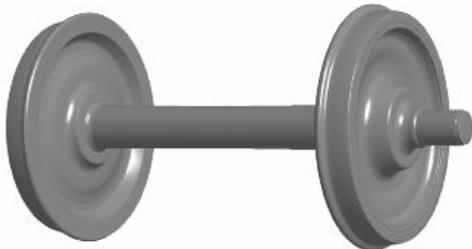
- Crack Growth
- Strain-Life
- Stress-Life

Specification:

Steel 1045, Q&T, BHN=336  
Steel 1045, Q&T, BHN=390  
Steel 1045, Q&T, BHN=410  
Steel 1045, Q&T, BHN=500  
Steel 1045, Q&T, BHN=563  
Steel 1045, Q&T, BHN=595  
Steel 300M, Su=1958.2  
Steel 4130 sheet, Su=1241.1  
Steel 4130 sheet, Su=806.7  
**Steel 4130, BHN=259**

### Steel 4130, BHN=259

<b>Technology</b>	Constant Amplitude Stress-Life
<b>Owner</b>	public
<b>Material Type</b>	steel
<b>Material Specification</b>	AISI 4130
<b>Material Alloy</b>	4130
<b>Brinell Hardness Number</b>	259
<b>Elastic Modulus</b>	E= 200000 MPa
<b>Ultimate Strength</b>	S <sub>u</sub> = 778 MPa
<b>Curve Intercept</b>	S <sub>f</sub> '= 1195 MPa
<b>Curve Slope</b>	b= -0.077
<b>Material Reference</b>	SAE J1099 - June 1998 (from eN data)



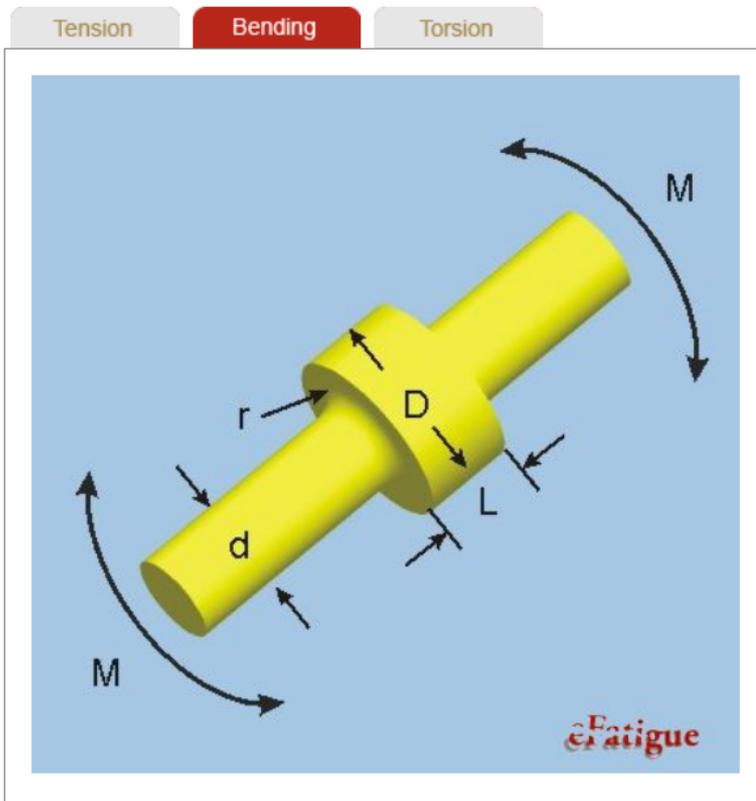
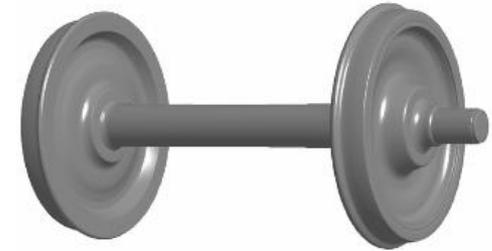
$$\sigma_a = \sigma'_f \cdot (2N)^b$$

$$\sigma_{FL} = 1195 \cdot (2 \cdot 10^7)^{-0.077} = 327.5 \text{ MPa}$$

# Example – continuation

<http://www.efatigue.com>

## Round Shaft with Double Fillets



### Variables

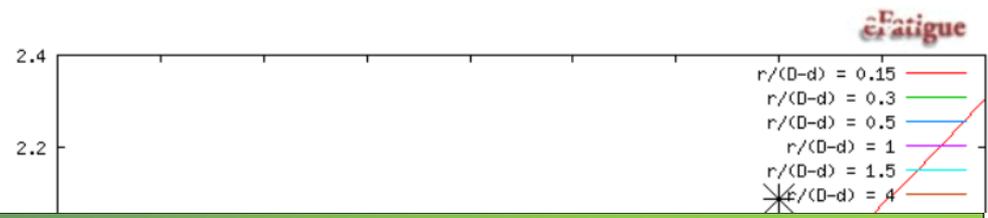
	Net Section Stress	▼
D	250	mm ▼
d	175	mm ▼
r	15	mm ▼
L	75	mm ▼

where  
 $0.01 < d/D$   
 $0.1 < r/(D-d) < 5$

### Results

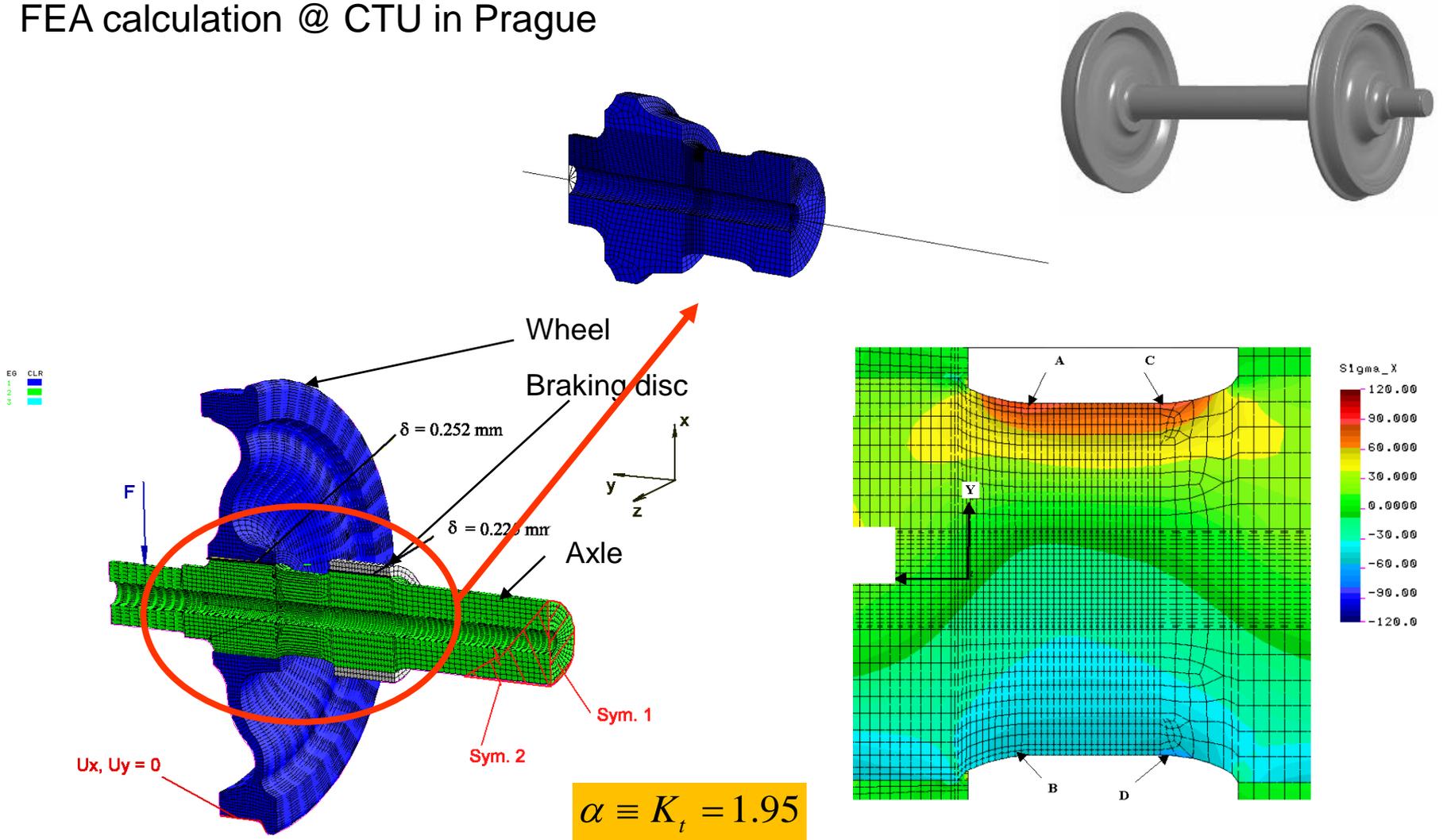
$K_t = 2.09$

### Peterson Plot

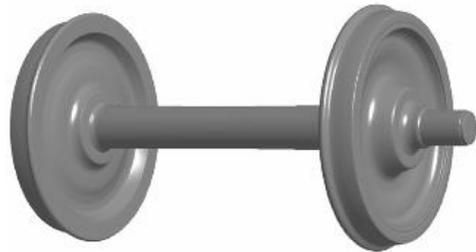


# Example – continuation

FEA calculation @ CTU in Prague

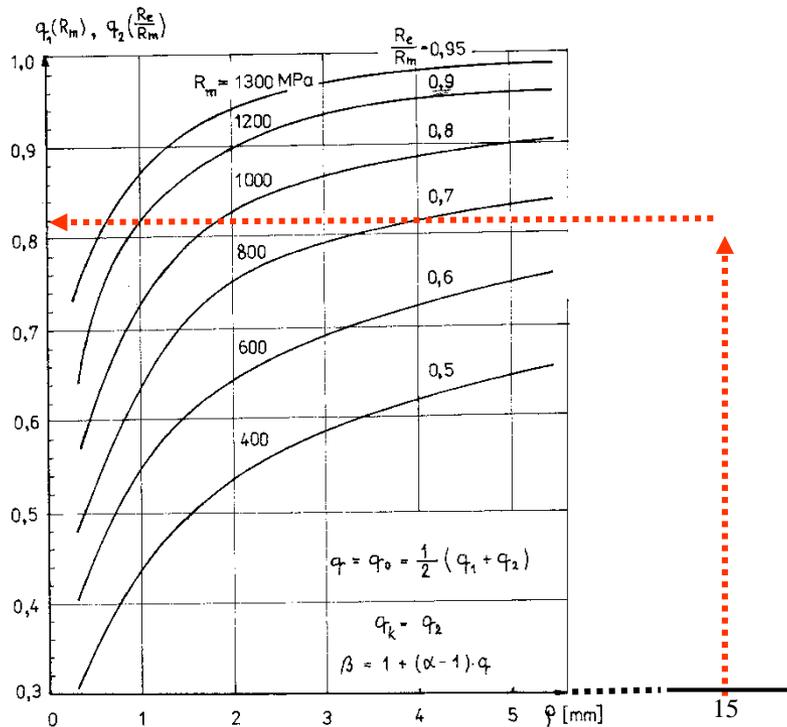


# Example – continuation



www.fadoff.cz

## Material - 25CrMo4



Mark	Value	Unit	Title	Group
A	12	%	Elongation at fracture	Static data
E	220000	MPa	Tensile elasticity modulus	Static data
f0	600	MPa	Fatigue limit	Repeated tension
f1	361	MPa	Fatigue limit	Fully reversed push-pull
f_d	34	mm	Diameter of specimen at active cross-section	Fully reversed push-pull
HV	332	-	Hardness (acc. to Vickers)	Static data
Ncf1	2000000	-	Number of cycles at fatigue limit	Fully reversed push-pull
RA	73	%	Reduction of area at fracture	Static data
sig_u	780	MPa	Ultimate tensile strength	Static data
sig_y	660	MPa	Tensile yield stress	Static data
T	20	degC	Ambient temperature	Static data
t1	228	MPa	Fatigue limit	Fully reversed torsion

$$q_1 \sim 0.83$$

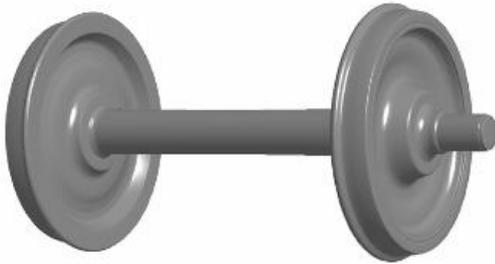
$$\frac{R_e}{R_m} = 0.85 \quad i.e.: q_2 \sim 0.93$$

$$\alpha \equiv K_t = 1.95$$

$$\beta = K_f = 1 + (K_t - 1) \cdot q = 1 + (1.95 - 1) \cdot (0.82 + 0.93) / 2 = 1.83$$

# Example – continuation

Estimation of the fatigue limit of a real part



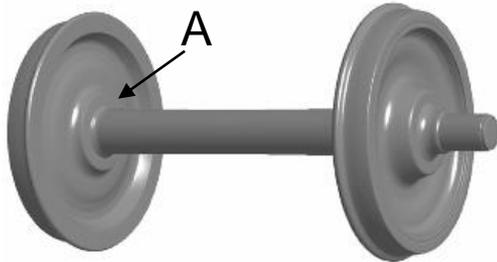
factor	$k$	value
loading	$k_L$	1.00
surface finish	$k_{SF}$	0.67
size factor	$k_S$	0.70
size factor	$k_T$	1.00

$$\sigma_{FL,N} = \frac{\sigma_{FL} \cdot k_L \cdot k_{SF} \cdot k_S \cdot k_T}{K_f}$$

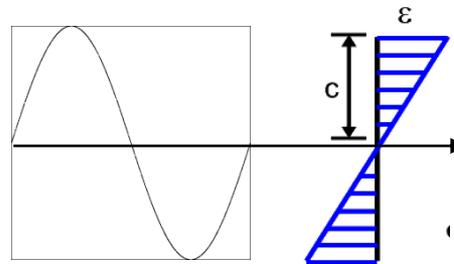
$$\sigma_{FL,N} = \frac{327.5 \cdot 1.00 \cdot 0.67 \cdot 0.70 \cdot 1.00}{1.83} = 83.9 \text{ MPa}$$

# Example – continuation

## Estimation of the nominal stress amplitude



Experimental strain amplitude measurement (in the point A):



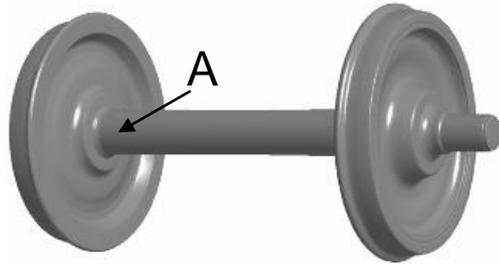
STRAIN

$$\varepsilon_{a,\max} = 312 \text{ [microstrain]}$$

$$\sigma_a = E \cdot \varepsilon = 220000 \cdot 0.000312 = 68.6 \text{ MPa}$$

# Example – continuation

Estimating the safety factor  $n_{FL}$



Loading:  $\sigma_a = 68.6 \text{ MPa}$

Structure properties:  $\sigma_{FL,N} = 83.9 \text{ MPa}$

$$n_{FL} = \frac{\sigma_{FL,N}}{\sigma_a} = \frac{83.9}{68.6} = 1.22$$



FKM-Guideline: Analytical Strength Assessment of Components in Mechanical Engineering. 5th revised edition. Frankfurt/Main, Forschungskuratorium Maschinenbau (FKM) 2003.

Table 4.5.1 Safety factors for steel \*<sup>3</sup> (not for GS) and for ductile wrought aluminum alloys ( $A \geq 12,5\%$ ).

j <sub>D</sub>		Consequences of failure	
		severe	moderate $\diamond^1$
regular inspections	no	1,5	1,3
	yes $\diamond^2$	1,35	1,2

$\diamond^1$  Moderate consequences of failure of a less important component in the sense of "non catastrophic" effects of a failure; for example because of a load redistribution towards other members of a statical indeterminate system. Reduction by about 15 %.

$\diamond^2$  Regular inspection in the sense of damage monitoring. Reduction by about 10 %.

# Questions and problems II

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1. What is the difference between the stress concentration factor and the notch factor? Write their relevant formulas.
2. Define the notch sensitivity factor of material and write its formula (as a function of a stress concentration factor and of a notch factor).
3. Is the stress concentration factor of metals a material parameter? And what about the notch factor?
4. Is the fatigue limit of a real part the same as the fatigue limit of a basic material? What other factors could be taken in the account by an expression of such fatigue limit?
5. Which shaft size results in a higher  $k_s$  size factor? Shaft with higher or smaller diameter?