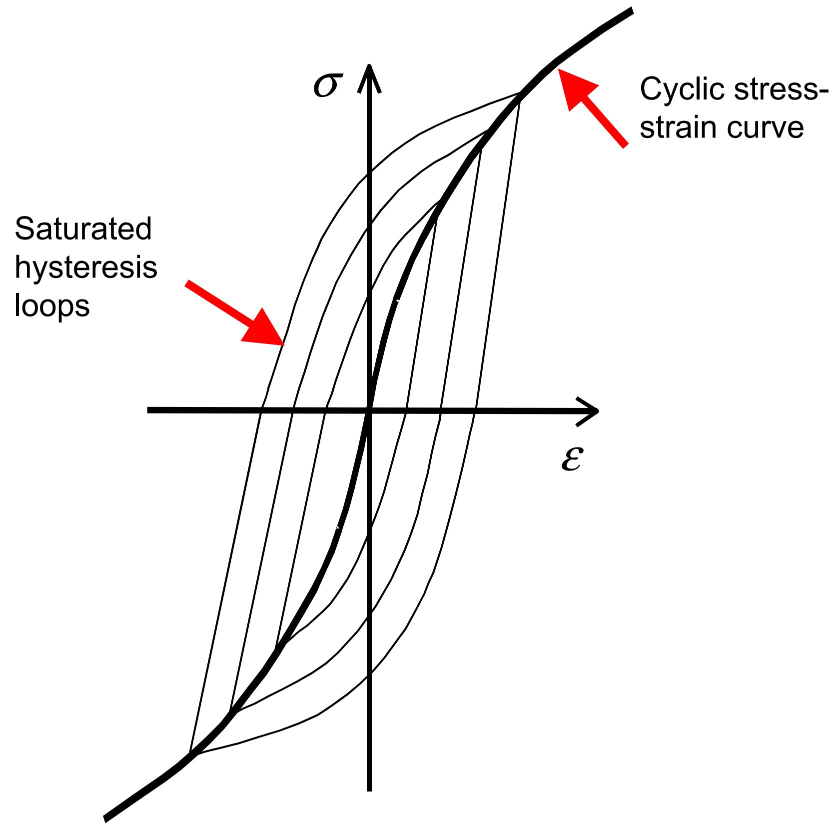
Dynamická únosnost a životnost

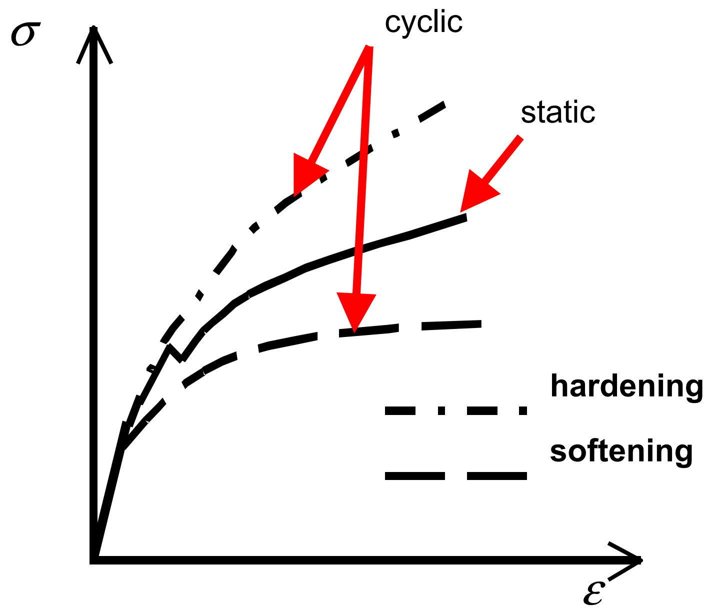
Lekce 4
-
Strain-Life Fatigue Analysis

Jan Papuga

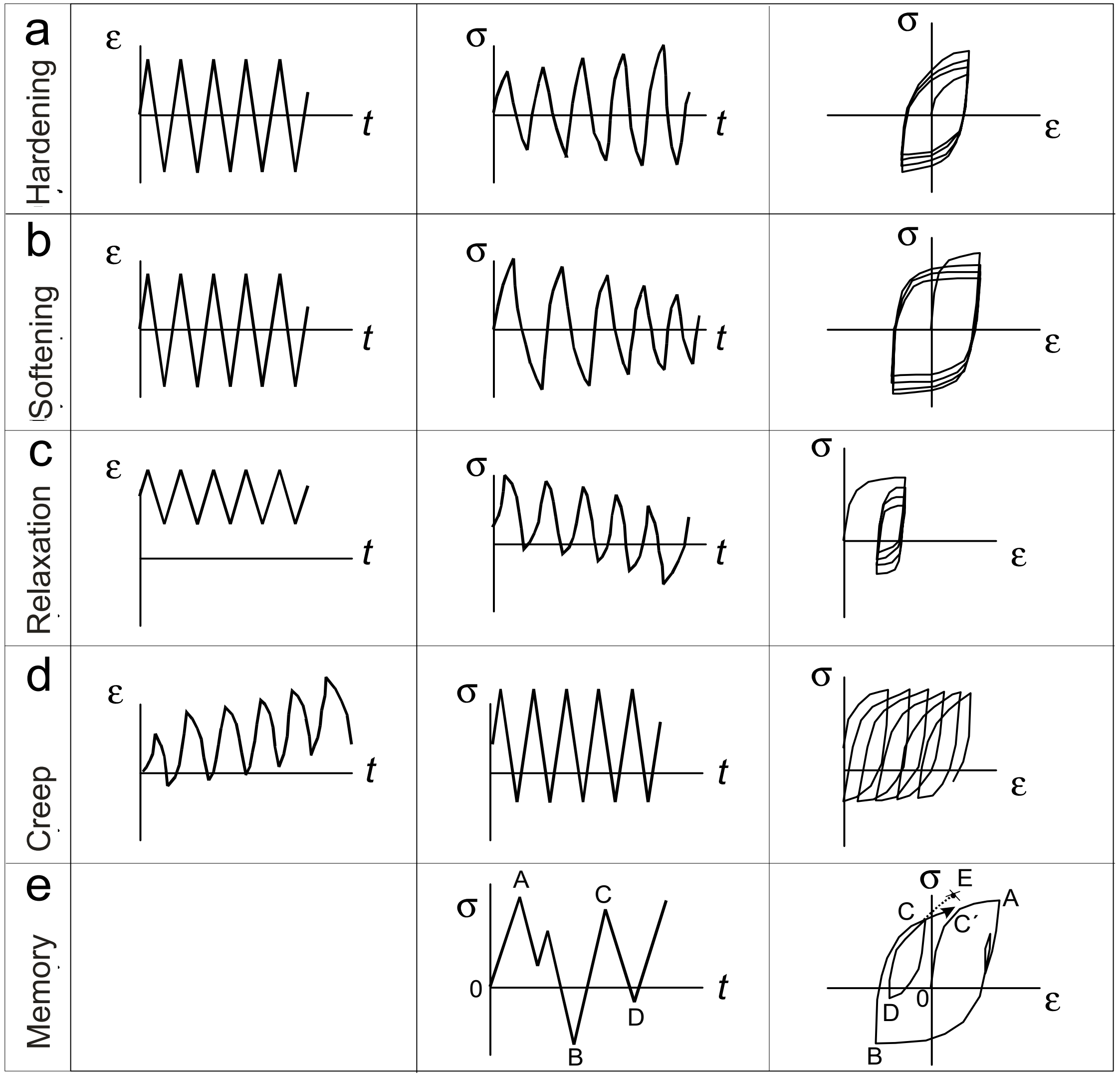
Strain-Life Analysis

Hysteresis loops, Cyclic stress-strain curve

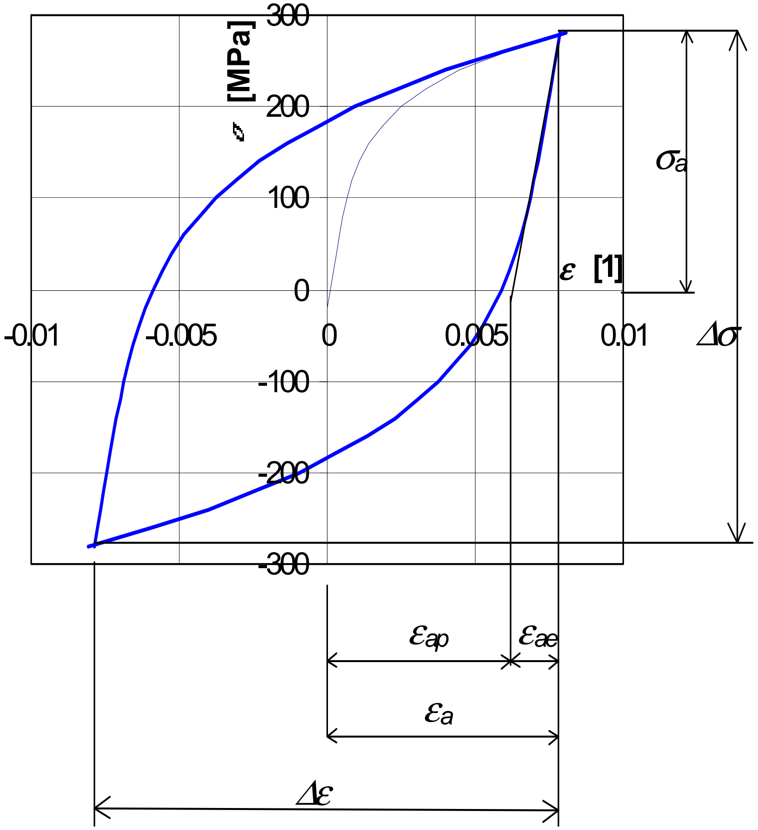




Material changes its characteristic during loading



CSSC approximation (Ramberg-Osgood)



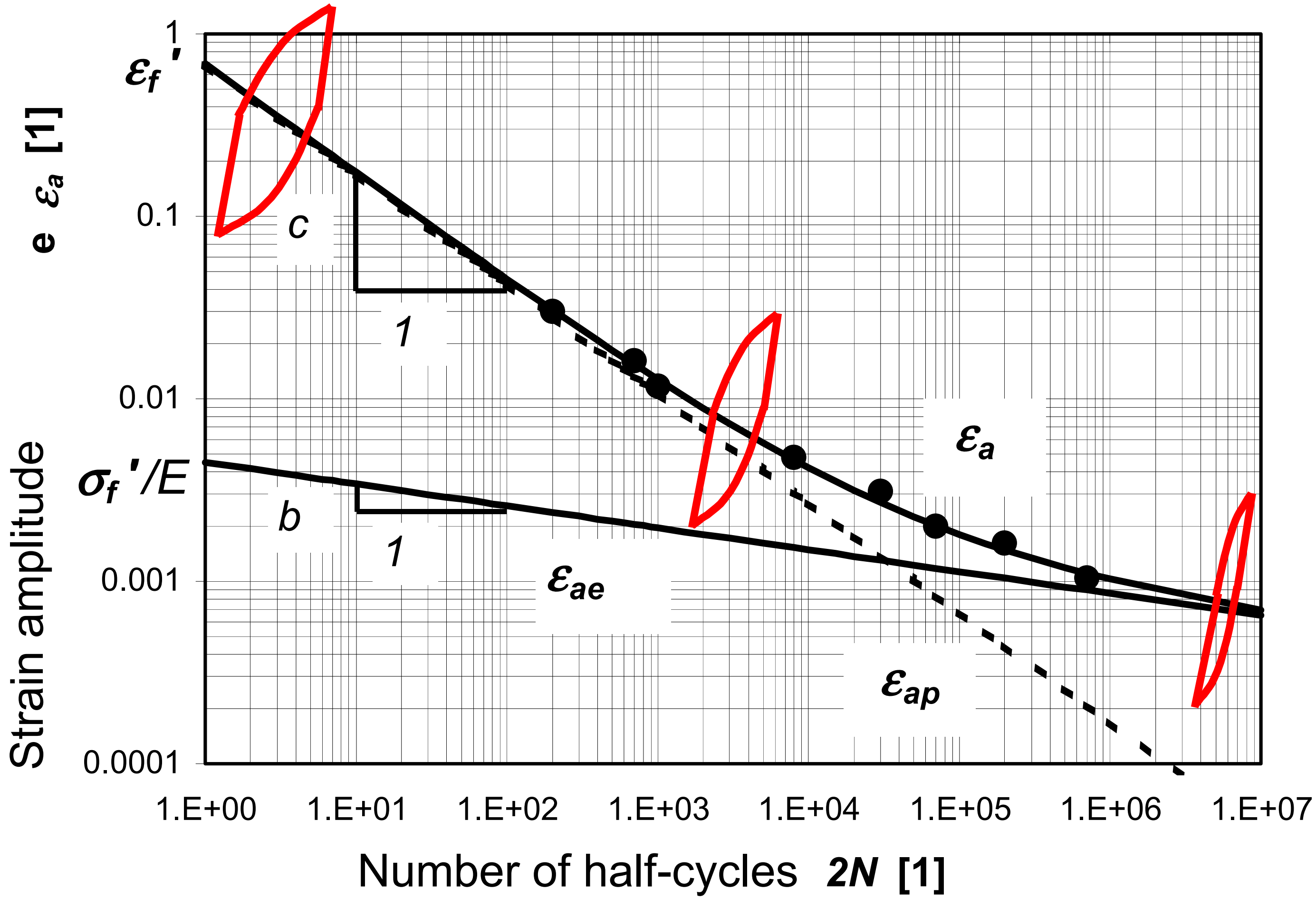
$$\sigma_a = K'\left(\varepsilon_a^{pl}\right)^{n'}$$

$$\varepsilon_a = \varepsilon_a^{el} + \varepsilon_a^{pl} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}}$$

K' - cyclic hardening modulus
n' - cyclic hardening exponent
E - Young's modulus

Strain-based fatigue curves

Manson-Coffin curve



Manson-Coffin – approximation

$$\varepsilon_a = \varepsilon_a^{el} + \varepsilon_a^{pl} = \frac{\sigma_f'}{E}(2N)^b + \varepsilon_f'(2N)^c$$

σ_f'…fatigue strength coefficient, b…fatigue strength exponent

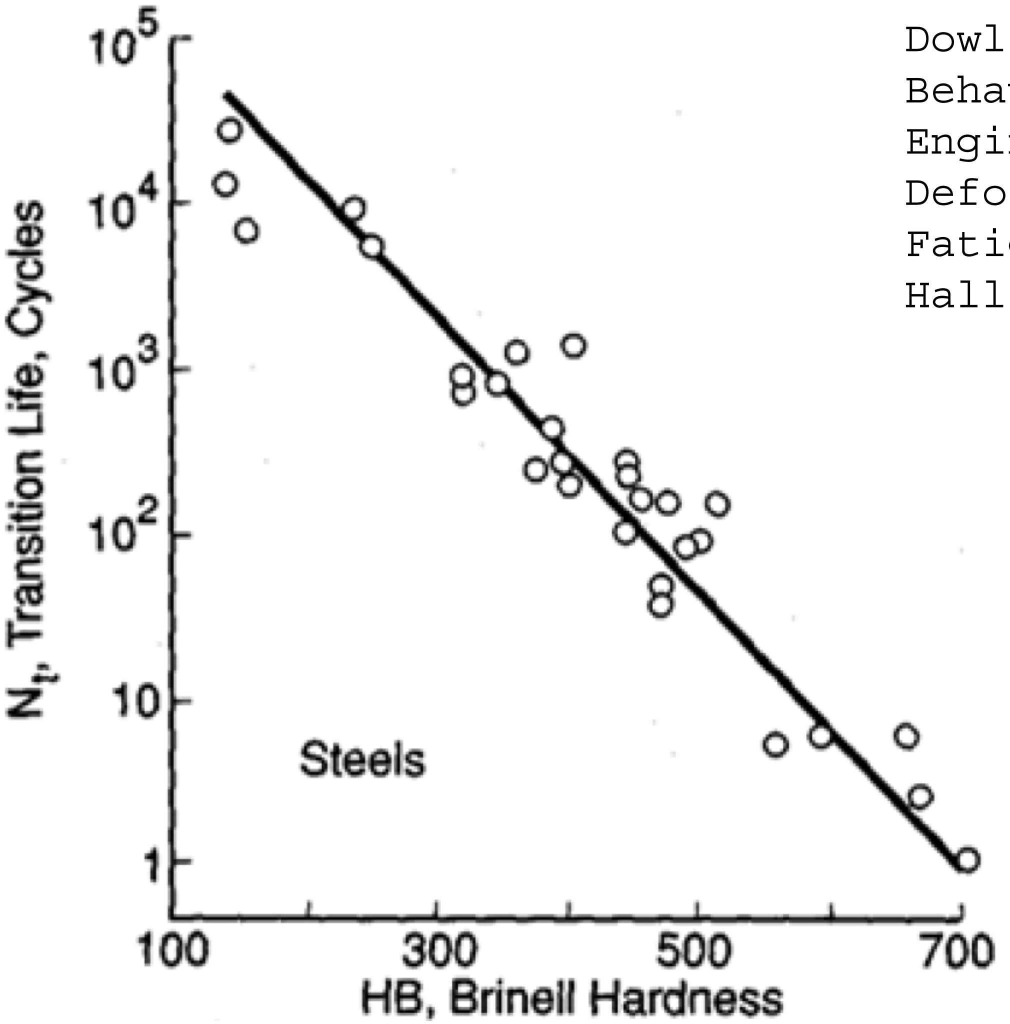
ε_f'… fatigue ductility coefficient, c…fatigue ductility exponent

$$\varepsilon_a^{el} = \frac{\sigma_f'}{E}(2N)^b, \qquad \varepsilon_a^{pl} = \varepsilon_f'(2N)^c$$

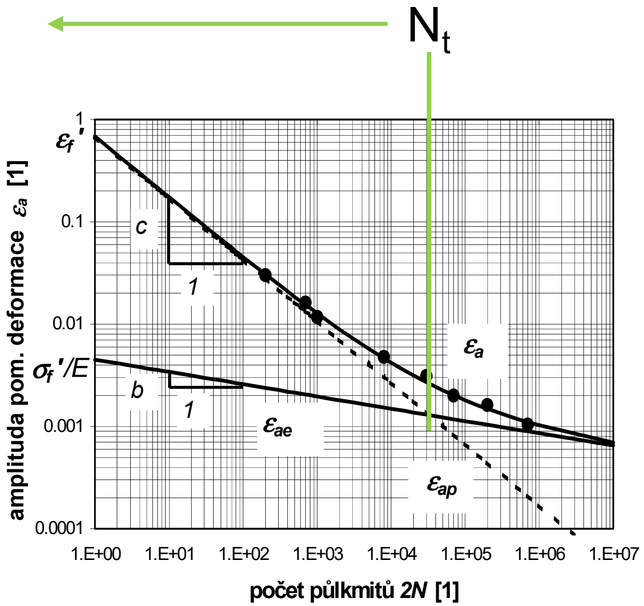
$$\log \varepsilon_a^{el} = \log \frac{\sigma_f'}{E}(2N)^b, \qquad \log \varepsilon_a^{pl} = \log \varepsilon_f'(2N)^c$$

$$\log \varepsilon_a^{el} = \log \frac{\sigma_f'}{E} + b \log(2N), \qquad \log \varepsilon_a^{pl} = \log \varepsilon_f' + c \log(2N)$$

Transition Fatigue Life

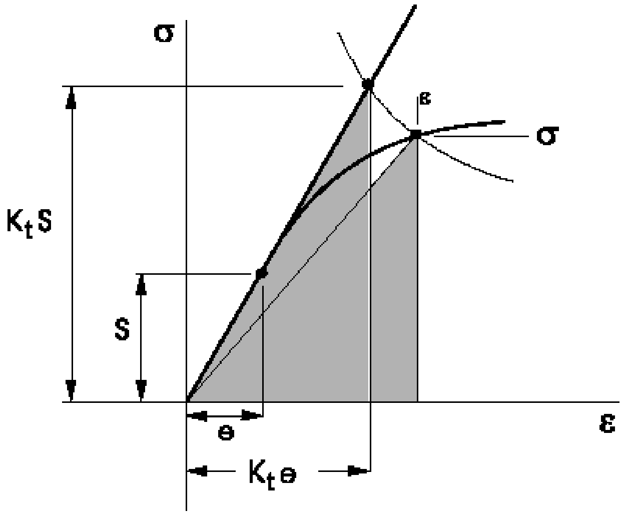


Dowling, N. E.: Mechanical Behavior of Materials: Engineering Methods for Deformation, Fracture and Fatigue. 2nd edition. Prentice Hall, Upper Saddle River 1999.



Neuber rule

Neuber

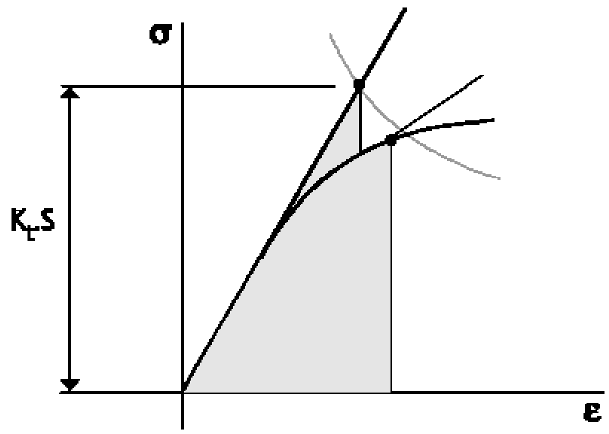


$$U_{fic} \overset{?}{=} U_V$$

$$U_{fic} = \boxed{\frac{1}{2}\sigma_{fic}\varepsilon_{fic} = U_v = \frac{1}{2}\sigma\varepsilon}$$

$$U_v = \frac{1}{2}\alpha^2 Se = \frac{\alpha^2 S^2}{2E} = \boxed{\frac{\sigma_{fic}^2}{2E} = \frac{\sigma}{2}\left[\frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}\right]}$$

Glinka



$$U_{fic} = \boxed{\frac{1}{2}\sigma_{fic}\varepsilon_{fic} = U_v}$$

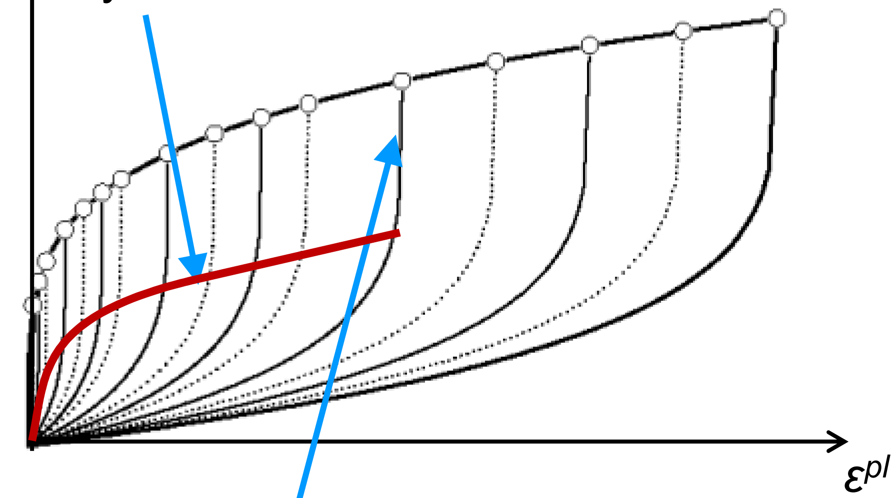
$$U_v = \frac{\sigma^2}{2E} + \int_0^{\varepsilon^{pl}} K'\left(\varepsilon^{pl}\right)^{n'} d\varepsilon = \boxed{\frac{\sigma^2}{2E} + \frac{\sigma}{n'+1}\left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}}$$

Material non-linearity - Consequencies

- **The loading by more load channels cannot be divided to the response of the FE-model to individual load cases**
- **Processing of long load histories can get quite lengthy**
  - **Either non-linear FEA**
  - **Or linear FEA, but then Neuber-like method should be applied to obtain the elastic-plastic response**

LCF – Masing Assumption

Cyclic stress-strain curve crosses centers of hysteresis curves shifted to their starts



$$\Delta\varepsilon = 2\varepsilon_a = 2\varepsilon_a^{el} + 2\varepsilon_a^{pl} =$$
$$= \frac{2\sigma_a}{E} + 2\left(\frac{2\sigma_a}{2K'}\right)^{\frac{1}{n'}} = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma_a}{2K'}\right)^{\frac{1}{n'}}$$

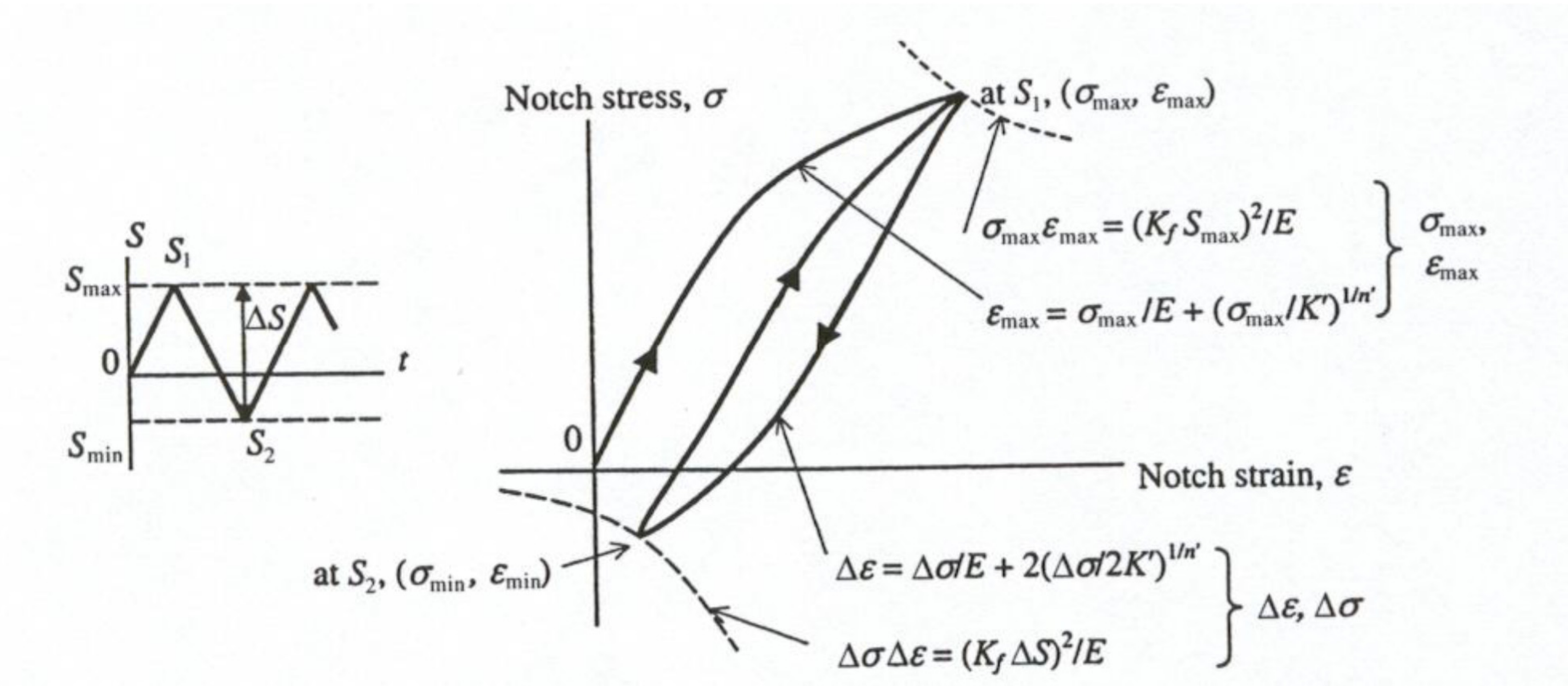
Massing assumes that saturated hysteresis loop (shifted in to the zero points of the coordinate system) have a common upper branch.

Many of metals, however, do not follow the rule.

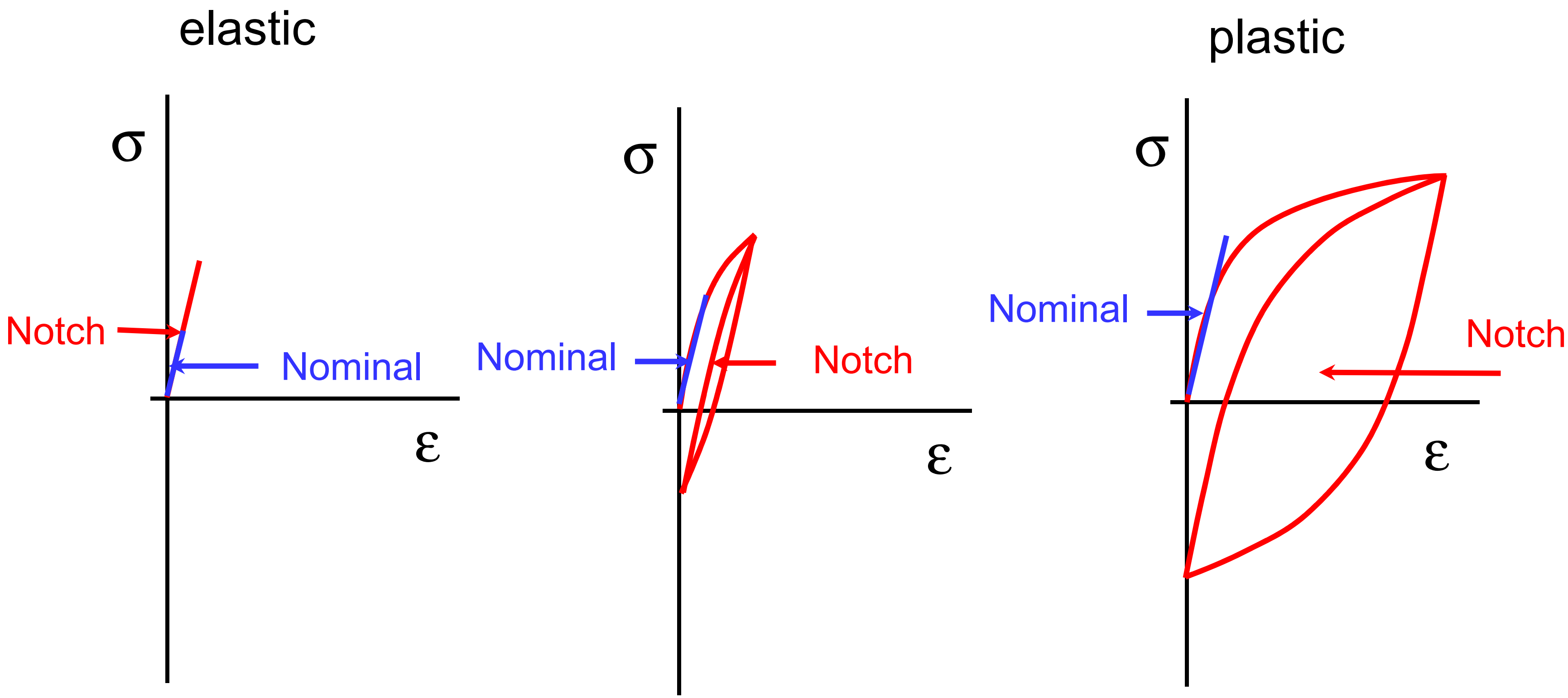
Real Neuber Rule Application

We should distinguish

- **First branch going from zero (A – according to cyclic stress-strain curve)**
- **Next branches correspond to the description of hysteresis loop**



Mean stress at notches



Nominal mean stress is less than notch mean stress

Nominal mean stress is greater than notch mean stress

MSE by Strain-Based Methods

Landgraf:

$$P_2 \equiv \varepsilon_{a,v} = \frac{\sigma'_f - \sigma_m}{E} \cdot (2N)^b + \varepsilon'_f \cdot (2N)^c$$

Material curve (Basquin) modified

Smith, Watson,Topper:

$$P_1 \equiv P_{SWT} = \sqrt{(\sigma_a + \sigma_m) \cdot \varepsilon_a \cdot E}$$

Damage parameter (load) modified

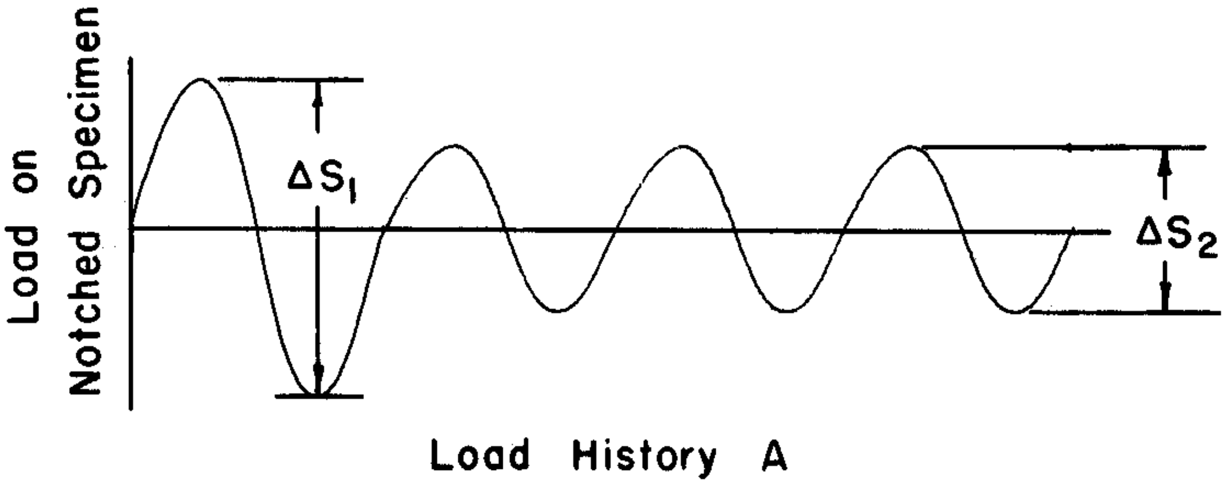
Bergmann:

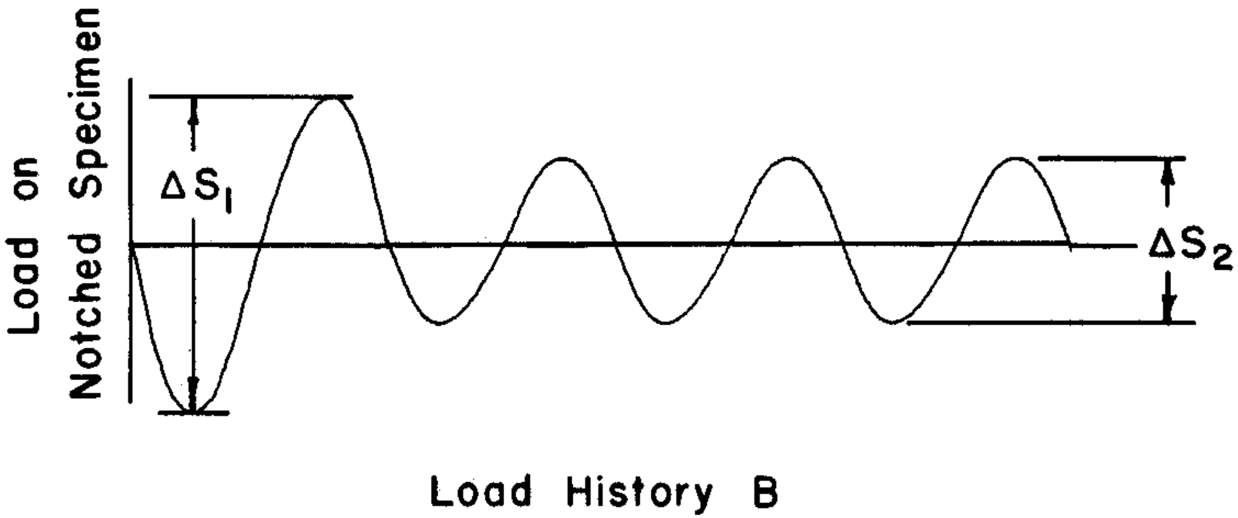
$$P_3 \equiv P_B = \sqrt{(\sigma_a + k \cdot \sigma_m) \cdot E \cdot \varepsilon_a}$$

Erdogan a Roberts:

$$P_4 \equiv P_{E,R} = \sqrt{\sigma_a^{\gamma} \cdot (\sigma_a + \sigma_m)^{1-\gamma} \cdot E \cdot \varepsilon_a}$$

Linearity of the damage cumulation rule

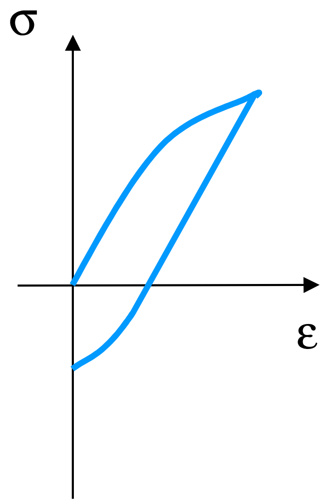


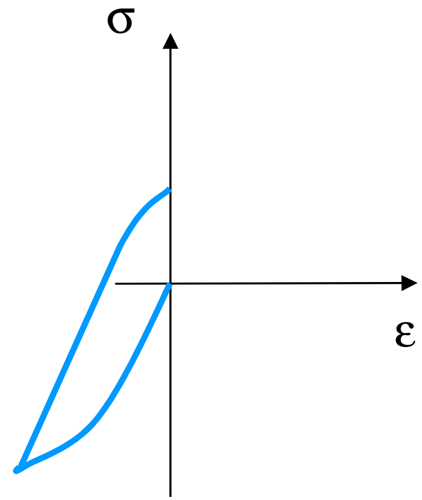


Will be the results the same?

Residual stresses

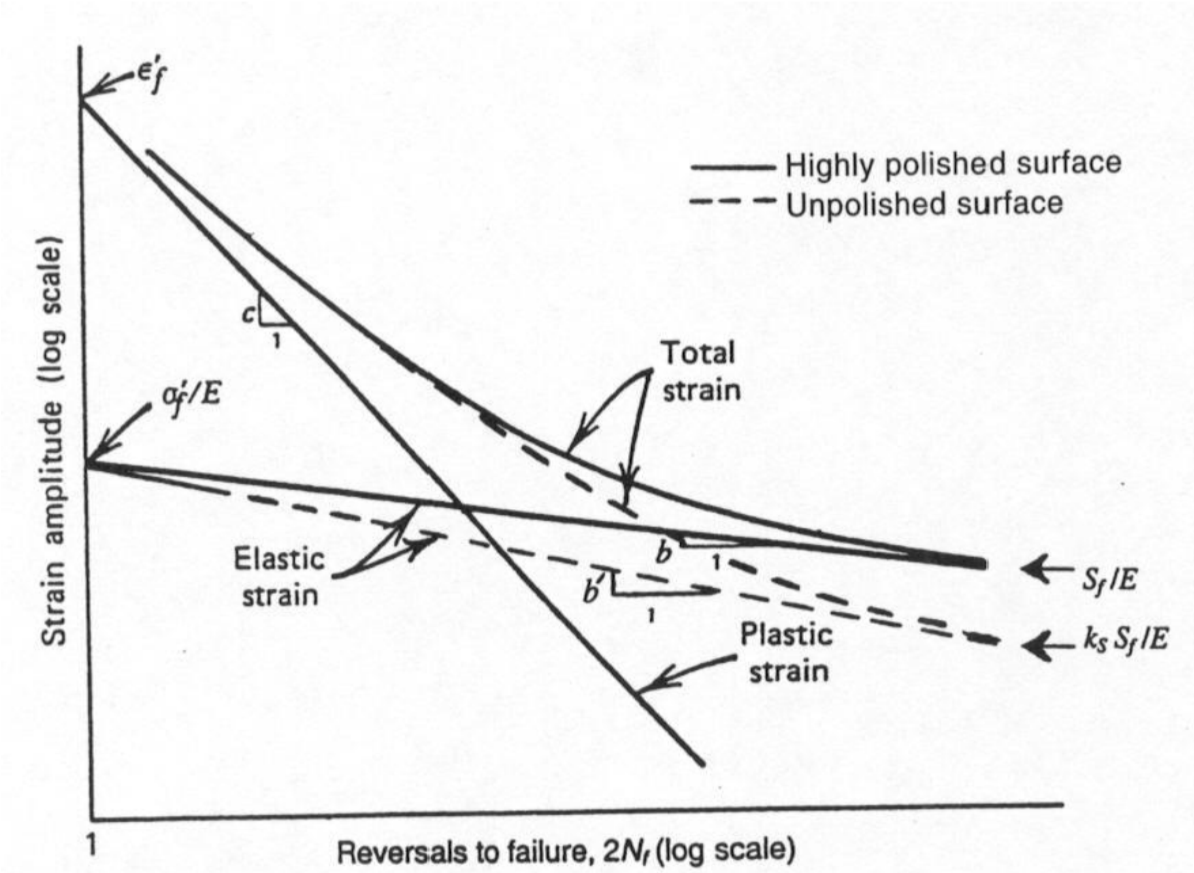
- **Result of previous history including plastic straining**
- **Compressive residual stress is positive (can be used in some technological procedures – autofrettage)**
- **Tensile residual stress worsens durability (e.g. too aggressive grinding)**





Surface roughness effect – e-N curves

- **Similar effect as by S-N curves**
- **The HCF region affected above all (i.e. the elastic part)**

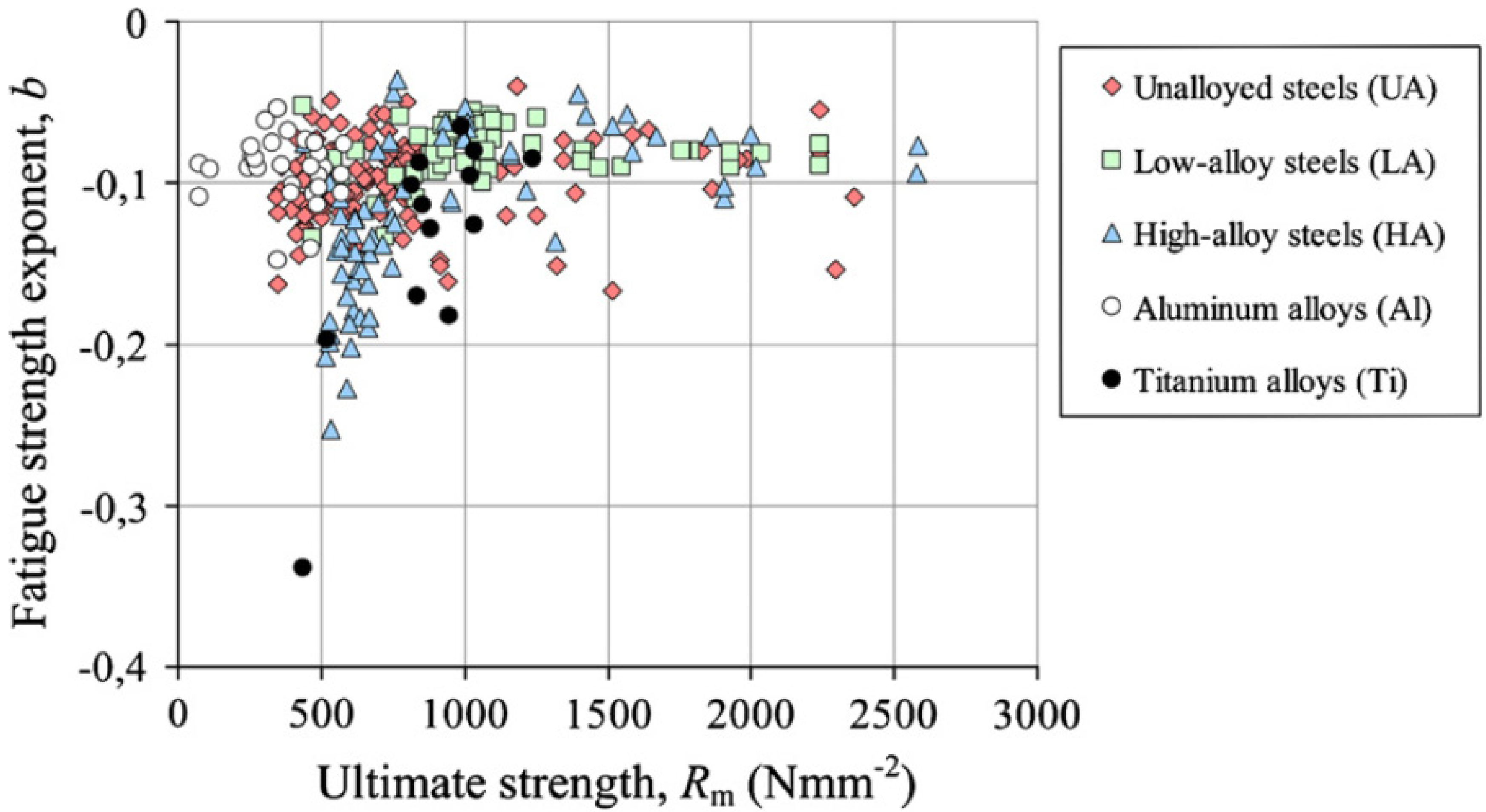


Fatigue parameters estimates

Parameter	Unalloyed and low-alloyed steels	Aluminum and titanium alloys
σ'_f	$1{,}5 \cdot R_m$	$1{,}67 \cdot R_m$
b	-0,087	-0,095
ε'_f	$0{,}59 \cdot \psi$	0,35
c	-0,58	-0,69
σ_C	$0{,}45 \cdot R_m$	$0{,}42 \cdot R_m$
ε_C	$0{,}45 \cdot \frac{R_m}{E} + 1{,}95 \cdot 10^{-4} \cdot \psi$	$0{,}42 \cdot \frac{R_m}{E}$
N_C	$5 \cdot 10^5$	$1 \cdot 10^6$
K'	$1{,}65 \cdot R_m$	$1{,}61 \cdot R_m$
n'	0,15	0,11
where: BÄUMEL, A.; SEEGER, T.: Material Data for Cyclic Loading - Suppl. 1. Materials Science Monographs *61*, Elsevier Sc. Publisher, Amsterdam 1990	$\psi = 1{,}0$ for $\frac{R_m}{E} \leq 3 \cdot 10^{-3}$ $\psi = \left(1{,}375 - 125{,}0 \cdot \frac{R_m}{E}\right)$ for $\frac{R_m}{E} > 3 \cdot 10^{-3}$ $\psi \geq 0$	

Another estimate: Manson (for steels) $\Delta\varepsilon = 2\varepsilon_a = 3{,}5 \frac{R_m}{E} \cdot (N)^{-0{,}12} + (\varepsilon_f)^{0{,}6} \cdot (N)^{-0{,}6}$

But carefully!



Basan R, Franulović M, Prebil I, Črnjarić-Žic N. Analysis of strain-life fatigue parameters and behaviour of different groups of metallic materials. Int J Fatigue 2011;33:484-491.

Design Against Fatigue
-
Ways to Fatigue Analysis, Part III

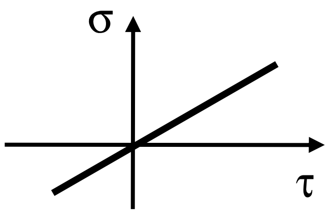
Jan Papuga

Typology of loading

Proportional loading

(in-phase loading)

The stress tensor in one moment is the multiple of a stress tensor in another moment

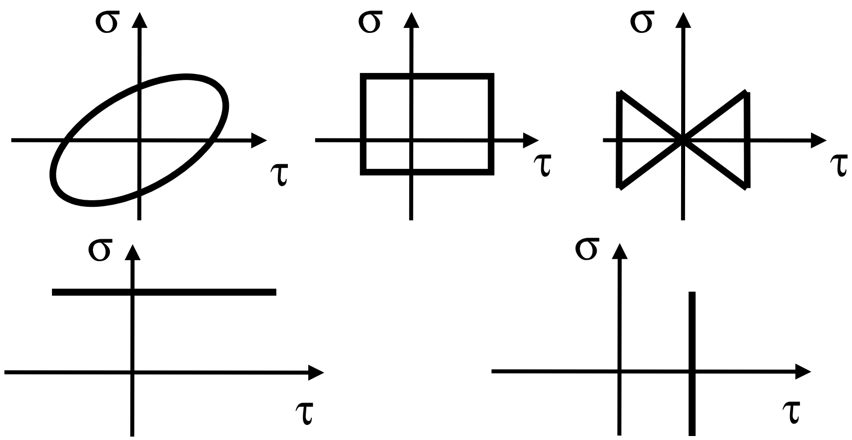


Non-proportional loading

(out-of-phase loading)

Change of individual components of the tensors does not correlate

Changes in principal directions occur – multiaxial hardening starts



One of the Simplest Solutions

Signed von Mises stress

- **Can be efficiently used also for loading with non-constant (or random) amplitude**

$$\sigma(t) = \sqrt{\frac{1}{2}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]}$$

PragTic $\sigma^*(t) = \sigma(t) \cdot sign\left(I_1(t)\right)$

MSC.Nastran (and PragTic) $\sigma^*(t) = \sigma(t) \cdot sign\left(\max\left(\left|\sigma_1\right|, \left|\sigma_3\right|\right)\right)$

$\Rightarrow$ σ_a, σ_m $\Downarrow$

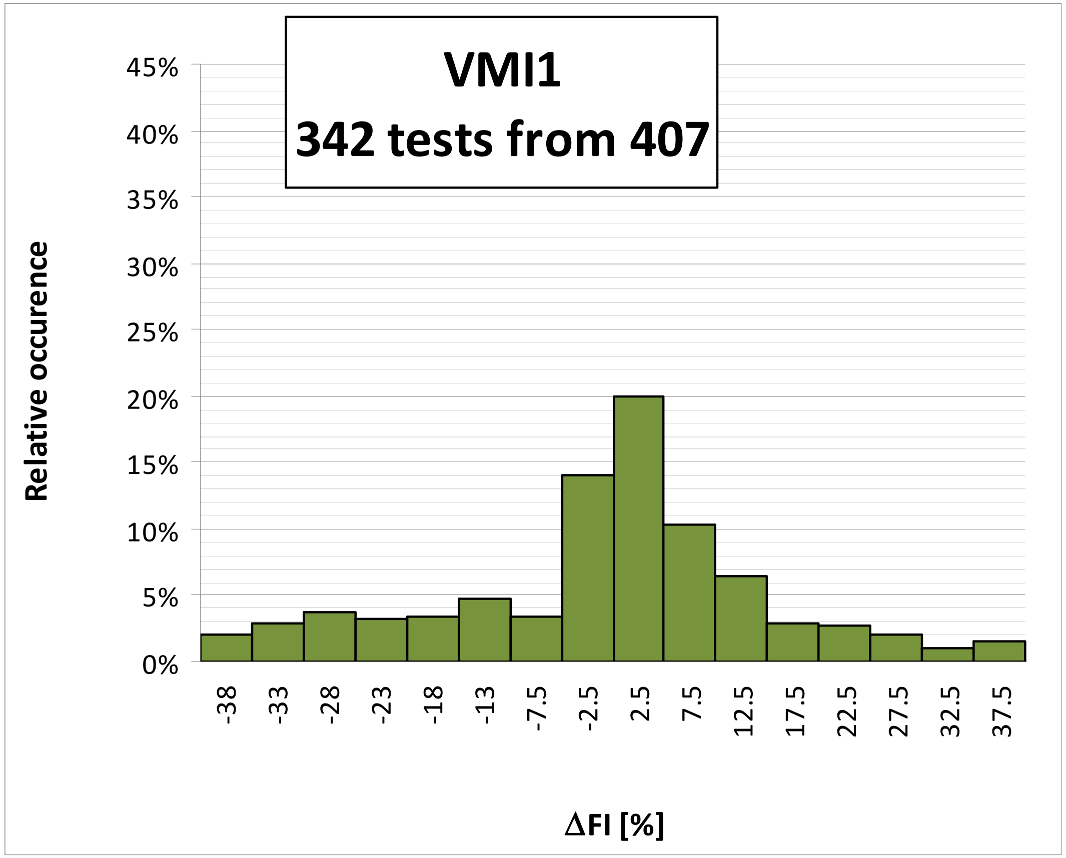
MSE e.g. by Walker

..But the Results…

Signed von Mises

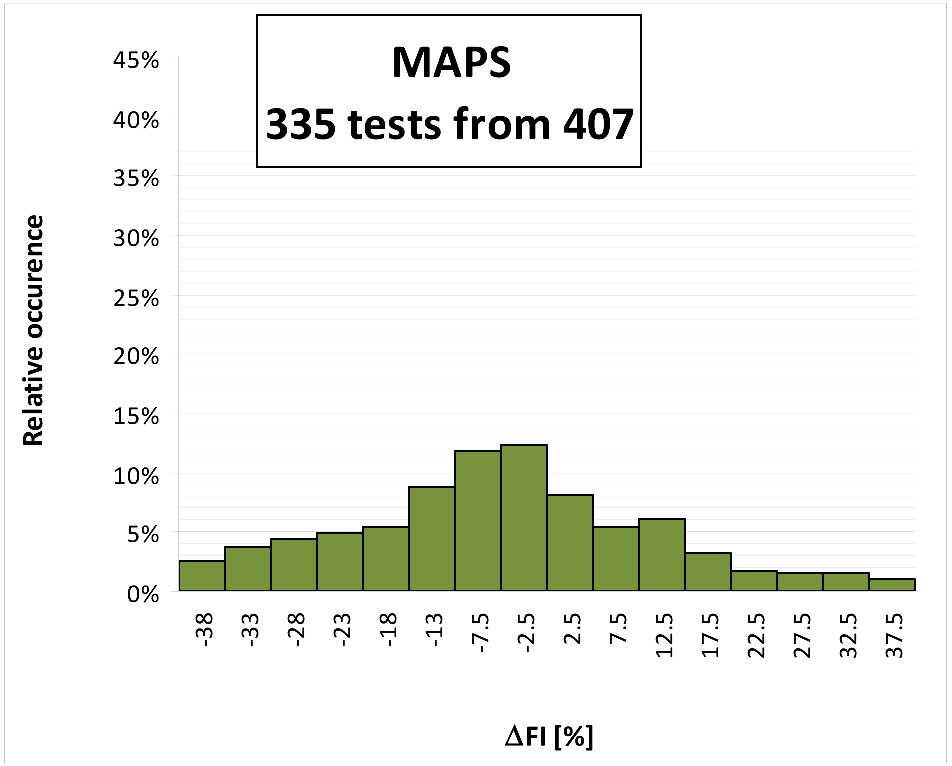
- Difference between both signing variants negligible overall
- Optimum variant only for ductile materials and in-phase loading with zero mean stress

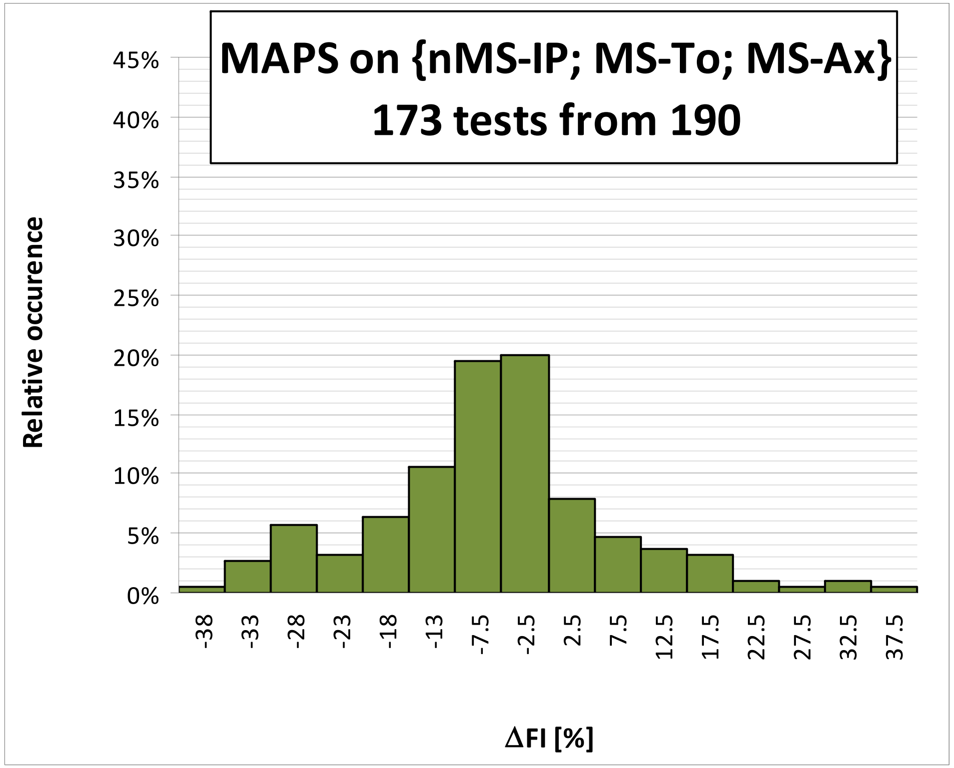
- Problems
  - Mean axial stress within multiaxial loading
  - Mean torsion stress within multiaxial loading



MSC.Fatigue - MAPS

Provides the best overall results if the simplified criteria are evaluated, but still they are not very good

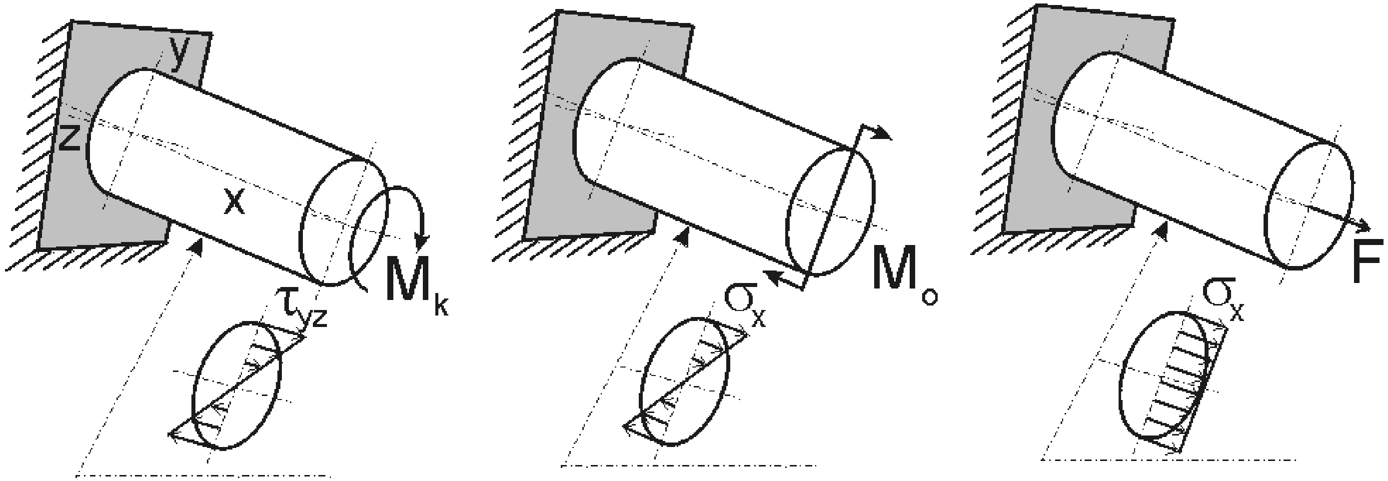




Multiaxial fatigue solution – Status quo

- Short development and verification time
- More interacting effects
- More complicated numerical analysis
- Unclear character of the damage initiation
- Too many existing solutions
- Their broader comparison missing (before FatLim)
- Major use in the research area only
- Demand on a universal solution exists in the commercial sphere

Surface



Maximum local loads are on the surface for any load mode.

This must be valid also for any their combination.

Exceptions:

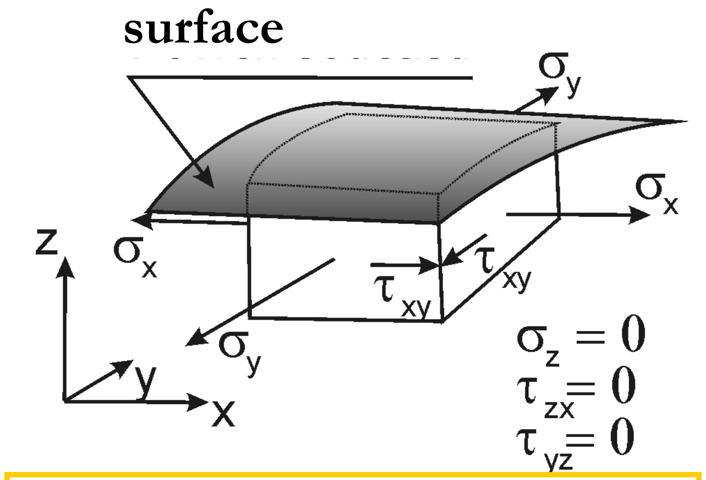
- **Bimaterial interface (also interface between hardened layer and matrix)**
- **Internal integrity defects, inclusions**
- **Forced on parts**
- **Contact surfaces**
- **Involved residual stresses**

Beware of GCF – crack needn't be initiated on the surface

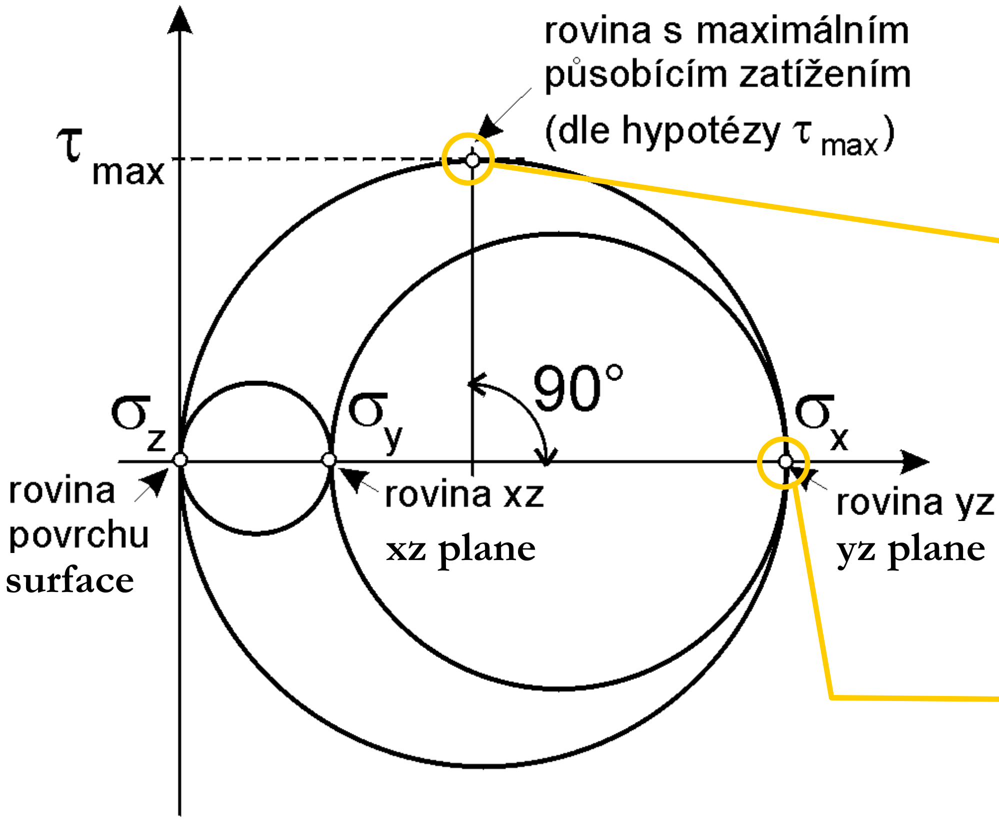
Damage Initiation on the Surface

Plane stress state :

(!not valid at contacts!)



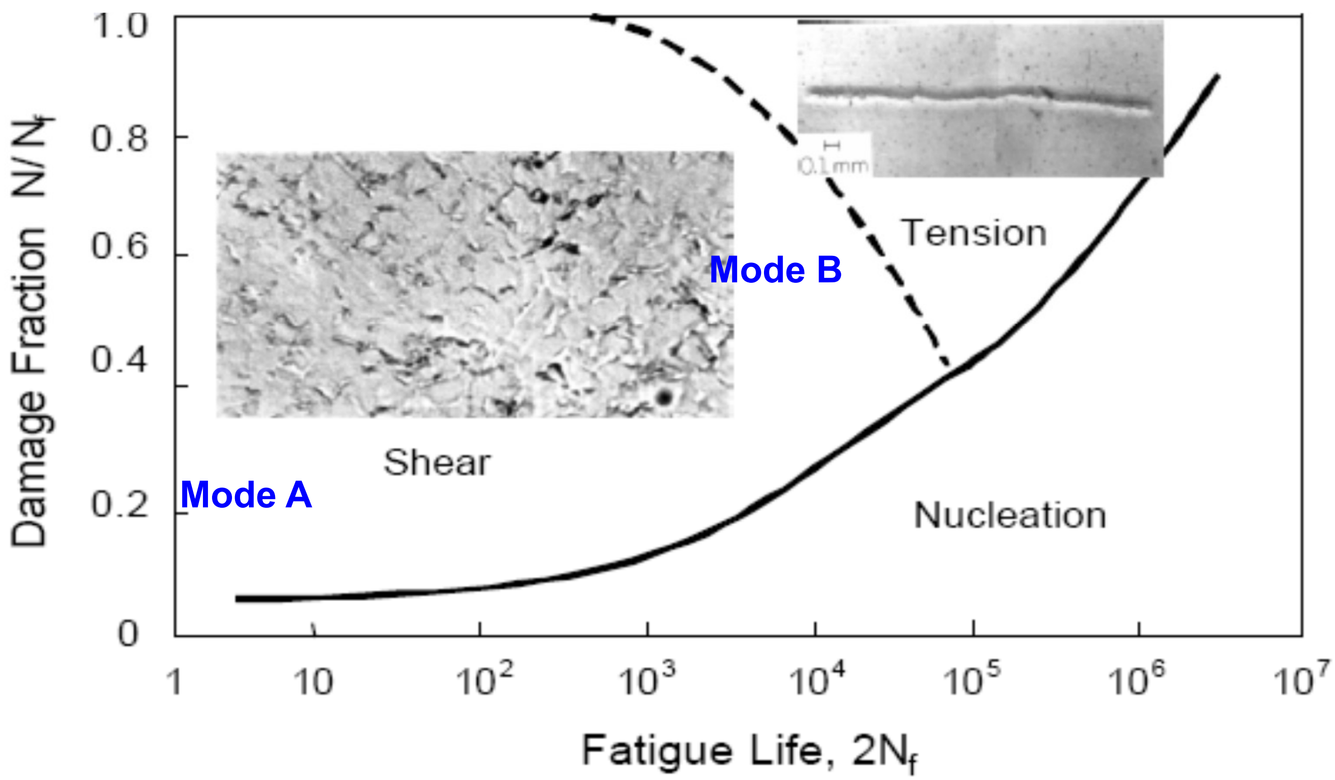
If σ_x and σ_y are principal stresses:



Maximum load according to τ_{max} (Tresca) hypothesis (deviated by 45° from the surface)

Maximum load as regards the maximum principal stress (deviated by 90° from the surface)

Damaging mechanisms



SOCIE, D. F.: Critical plane approaches for multiaxial fatigue damage assessment. In: Advances in Multiaxial Fatigue, ASTM STP 1191. Red. D. L. Dowell a R. Ellis, Philadelphia, American Society for Testing and Materials 1993, pp. 7-36.

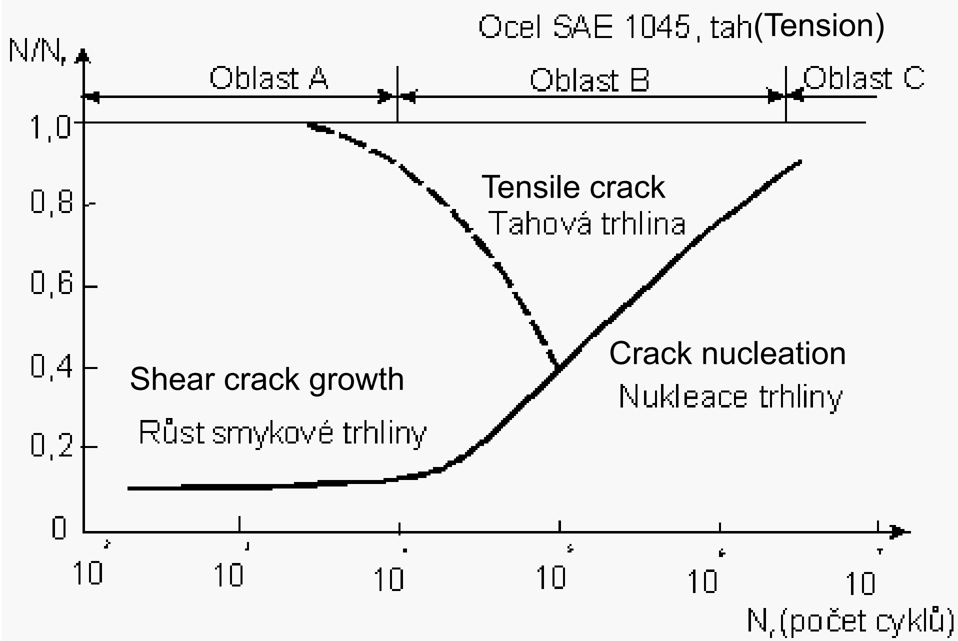
Cracking Mode

Region A – low-cycle fatigue, shear plastic straining dominant
 Plethora of micro-cracks initiated on shear planes; they become more highlighted but do not grow further

Region B – nucleation in the shear mode, then the macro-crack grows perpendicularly to the direction of the maximum principal stress.
 The nucleation is a small part of the total life. Next damage ascribed to the growth of the major crack. Some material can damage in this mode preferably.

Region C – high-cycle fatigue. The most of the life spent on a crack nucleation (in the shear mode). The macro-crack grows fast in the normal mode.

- Depends on:
  - Material
  - Load type

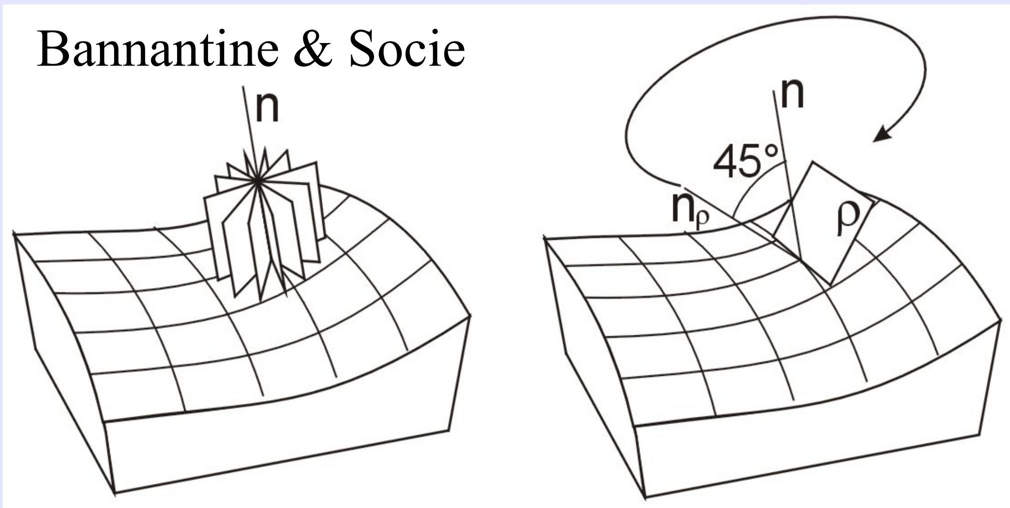


Nucleation of fatigue cracks in the shear mode dominant

Multiaxial Fatigue Calculation

Common Assumptions

- **Induced damage ~ Crack ~ Plane**
- **Load state description on that plane constitutes direct input for fatigue parameter**
- **The shear parameters play usually the dominant role, normal parameters are secondary (but important anyhow)**

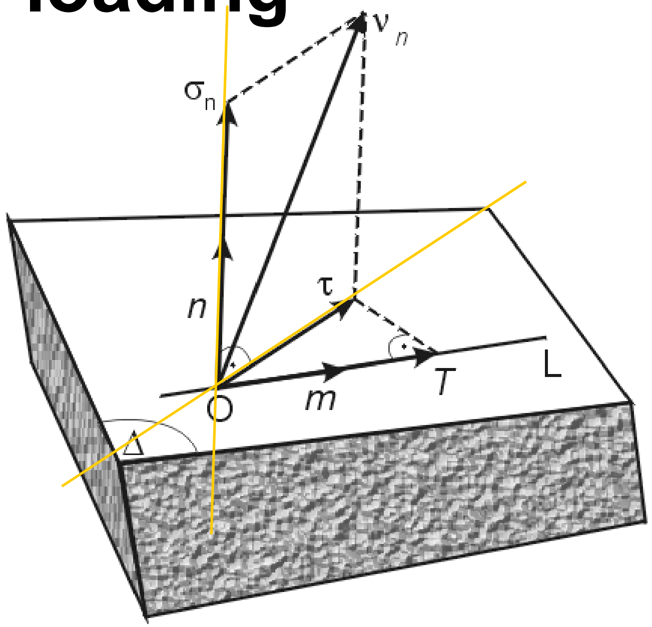


For reflection:

- If the FEM model is used, minor details are often modelled with abrupt changes of the surface plane.
- Is the plane stress condition valid there?
- How to define the normal line to the surface?

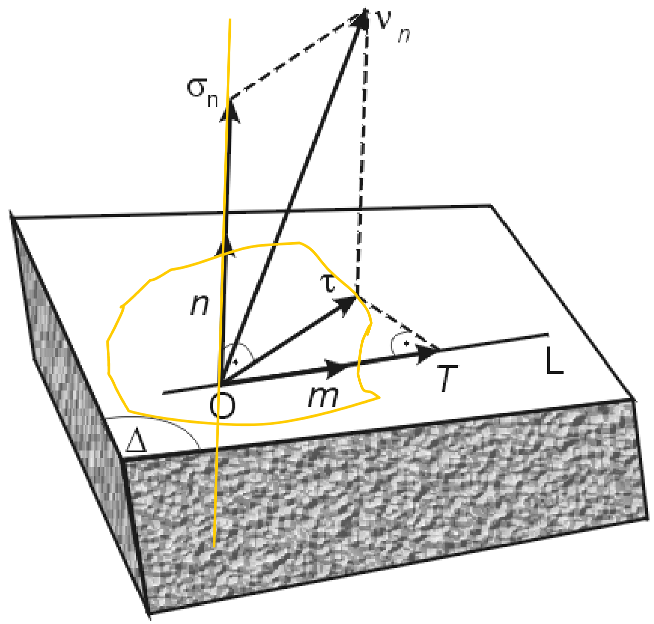
Stress State on a Plane

Proportional loading



- Directions of σ_n, ν_n and τ remain the same during loading

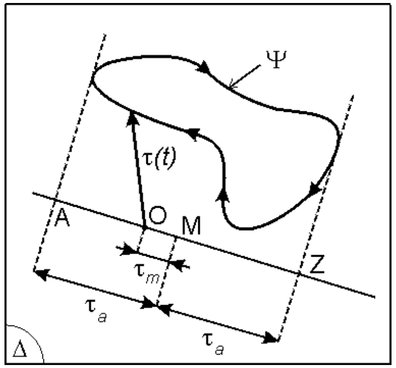
Non-proportional loading



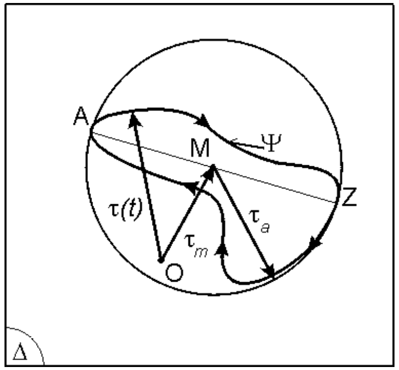
- Directions of σ_n remain the same during loading
- Shear stress can rotate

Shear Stress Decomposition

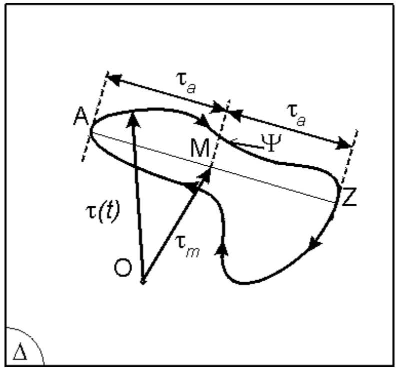
- How to define shear stress amplitude and mean stress by non-proportional loading?



Longest Projection Method

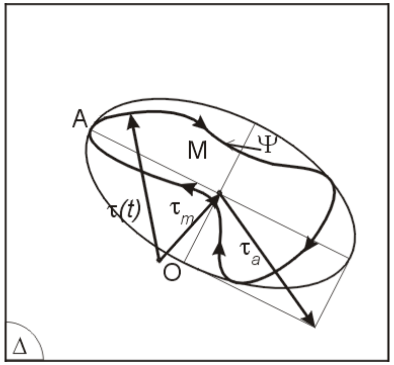


Minimum Circumscribed Circle Method



Longest Chord Method

? Isosceles ?

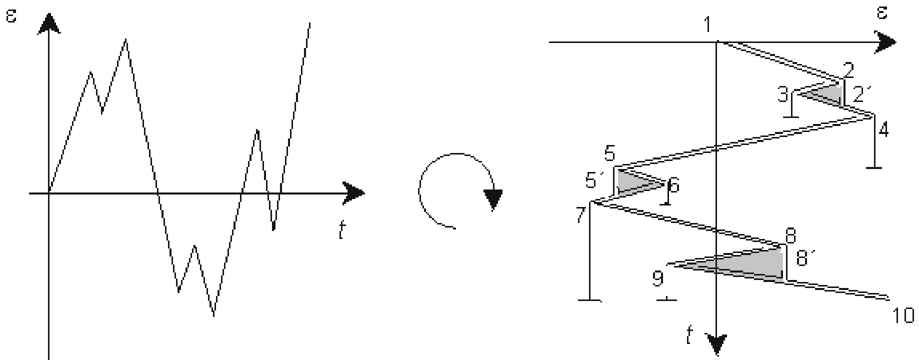


Minimum Circumscribed Ellipse Method

Well, but what is the cycle?

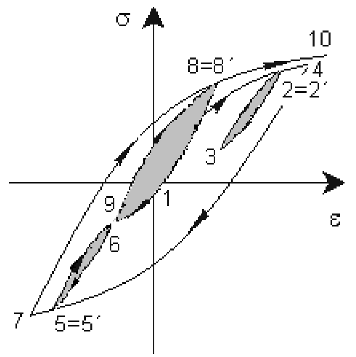
Load History Decomposition I

- Continuous load history -> Set of discrete cycles



To znamená, že jsou nalezeny následující položky:
Horní<tab>Dolní<tab>Počet

2	3	1.0
6	5	1.0
8	9	1.0
1	4	0.5
4	7	0.5
7	10	0.5

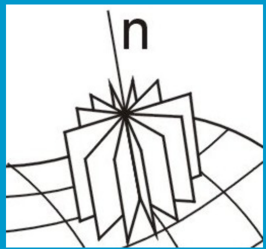


- What to do in the case of multiaxial non-proportional loading?

Load History Decomposition II – Multiax.

- Why not to use the rain-flow procedure?
  - on normal stress (Langlais, Vogel, Chase and others)
  - on resolved shear stress (Wang & Brown, Socie)
  - on some other equivalent parameter (Kenmeugne et al.)
- Well, why to use it?
  - The idea behind the rain-flow was based on the energy closed within the complete hysteresis loop
  - Where are the closed loops here?
  - Selection of some variable ensures omission of the effect of others
- More complicated models
  - Wang & Brown decomposition as proposed in 1996
  - No decomposing at all? Continuous damaging? Stefanov's method:
    - http://www.freewebs.com/fatigue-life-integral/

Critical Plane Approach



Compute X_i on all planes

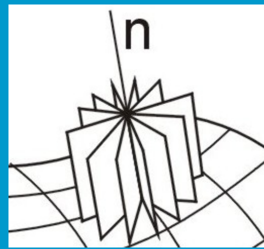
$$CP = P(\max_i(X_i))$$

$$D = D_{CP}$$

Mc Diarmid, Wang & Brown, Socie

- Critical plane according to:
  - Maximum Shear Stress Range (MSSR)
  - Maximum Damage (MD)
  - Critical Plane Deviation (CPD)
  - other...

Integral Approach



Compute X_i on all planes

$$X = \operatorname{aver}_i (X_i)$$

$$D = f(X,\ldots)$$

Papadopoulos, Kenmeugne et al.

- Averaging ~ Integration
- Integrate:
  - Complete damage parameter
  - Individual variables

Critical Plane Approaches

- MSSR (maximum shear stress amplitude)
  - One large shear stress amplitude on one plane can create less damage than a large number of smaller cycles elsewhere
- MD (maximum damage)
  - This approach should overcome the problem above
  - More actual

Multiaxial solution

- Solution enclosing the load path in the Ilyushin deviatoric space (IDS)
- Critical plane approach – CPA:
  - Final damage is related to the maximum of the damage parameter
  - Damage
    - localized to one specific plane
    - has no relation to a damage computed on any other plane
- Integral approach – IA:
  - The damage parameter or individual variables are integrated (i.e. averaged) over all planes
  - Damage
    - is not isolated to one specific plane
    - even perpendicular planes interact
    - the extremes found on particular planes are suppressed

IDS methods

$$\boldsymbol{\sigma} = \mathbf{s} + \mathbf{K}_\sigma$$

$$\mathbf{s} = \begin{bmatrix} \frac{2}{3}\sigma_x - \frac{1}{3}\sigma_y - \frac{1}{3}\sigma_z & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & -\frac{1}{3}\sigma_x + \frac{2}{3}\sigma_y - \frac{1}{3}\sigma_z & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & -\frac{1}{3}\sigma_x - \frac{1}{3}\sigma_y + \frac{2}{3}\sigma_z \end{bmatrix}$$

$$\mathbf{K}_\sigma = \frac{1}{3}\begin{bmatrix} \sigma_x + \sigma_y + \sigma_z & 0 & 0 \\ 0 & \sigma_x + \sigma_y + \sigma_z & 0 \\ 0 & 0 & \sigma_x + \sigma_y + \sigma_z \end{bmatrix}$$

Hydrostatic stress:

$$\sigma_H = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

IDS ~ Ilyushin Deviatoric Space – five-dimensional space, in which the point position is derived from stress tensor deviator

$$s_1 = \sqrt{\frac{3}{2}} s_{xx}; \qquad s_2 = \frac{1}{\sqrt{2}}\left(s_{yy} - s_{zz}\right);$$

$$s_3 = \sqrt{2} s_{xy}; \quad s_4 = \sqrt{2} s_{xz}; \quad s_5 = \sqrt{2} s_{yz}.$$

$$\sqrt{J_2} = \sqrt{\frac{1}{2}\mathbf{s}\cdot\mathbf{s}} = \sqrt{s \cdot s}$$

IDS – Why to be interested

Critical plane methods and integral criteria

- analyze stress state on various planes
- to get the shear stress range, the minimum circumscribed circle algorithm (MCCM) is run in 2D on each plane

IDS methods

- the same MCCM algorithm is run in 5D
- but only once!
- results in a quicker solution

Criteria for Fatigue Limit Estimation

How to check them?

- **All fatigue criteria converted to the standard:**

$$D_p \leq f_{-1}$$

f_{-1}- fatigue limit in fully reversed axial loading

- For an experimentally set multiaxial fatigue limit:

$$D_p = f_{-1}$$

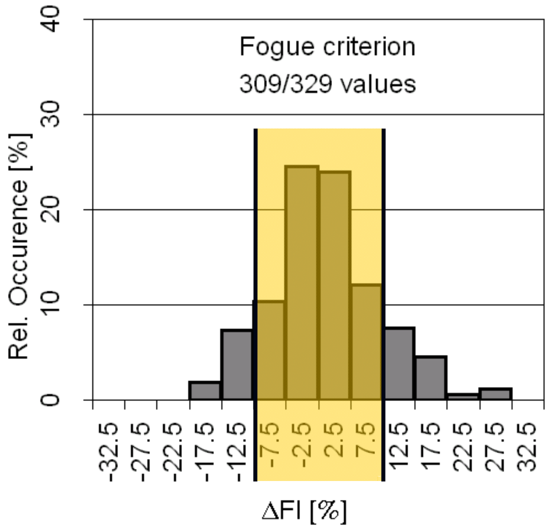
! f_{-1} – type of axial loading (push-pull, bending, rotating bending) corresponds to the fatigue limit used in calculation

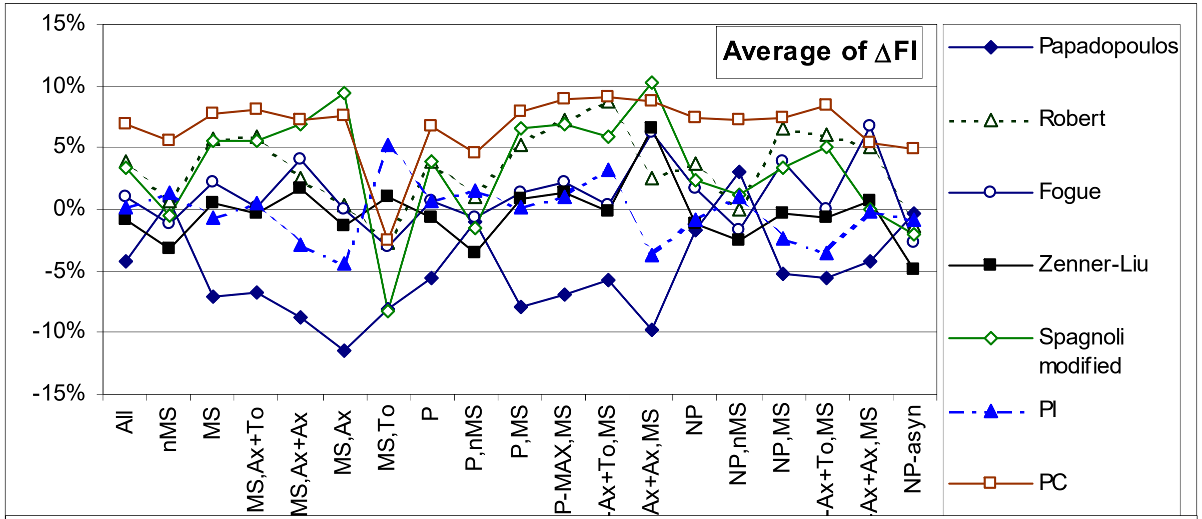
- Fatigue index error:

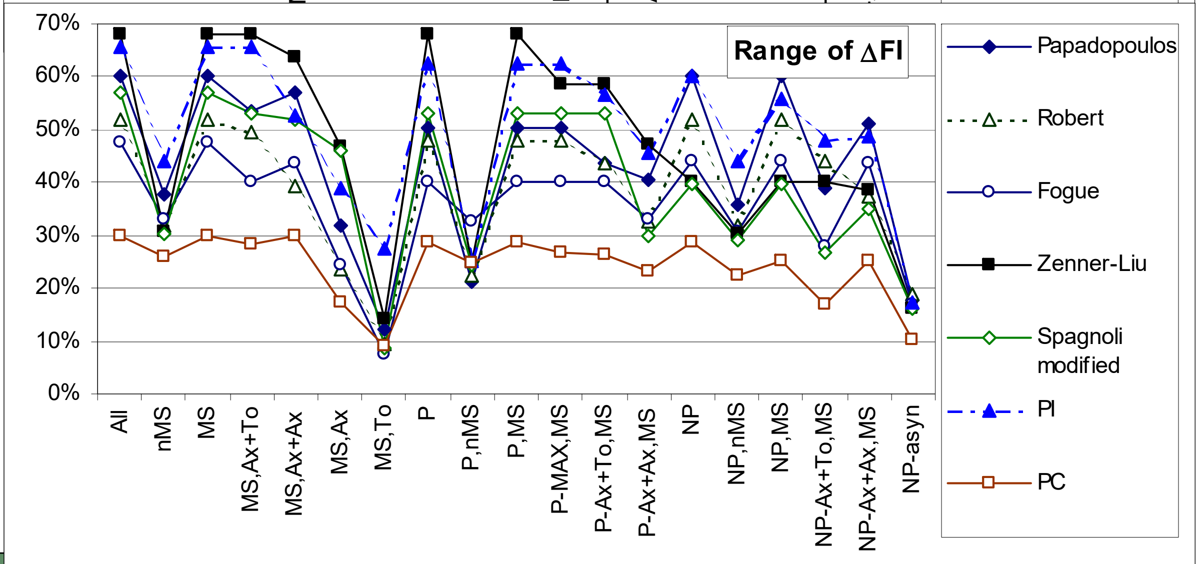
$$\Delta FI = \left(\frac{D_P - f_{-1}}{f_{-1}} \right) \cdot 100\%$$

FL Criteria - Method of Evaluation II

- **Statistical evaluation of D*FI* within prepared groups**
  - **average value**
  - **range (max-min)**
  - **standard deviation**
- Histograms of fatigue index error







Fatigue limit solution today

In fatigue solvers:

Fatigue limit estimation tools in available fatigue solvers	commercial							non-commercial			validation on 407 exps.	
	Fe-Safe	MSC.Fatigue	Femfat	nCode Design Life	FEARCE	LMS Virtual Lab C. Durability	WinLIFE	eFatigue	Code Aster	PragTic	mean ΔFI	standard deviation of ΔFI
Dang Van (1973, 1989)	X	X		X	X	X	X	X	X	X	-0.1%	12.2%
Findley (1957)							X	X		X	8.7%	15.2%
McDiarmid (1991, 1994)		X		X	X					X	-6.2%	12.0%
Sines (1959)								X		X	-4.3%	17.9%
Matake (1977)									X	X	6.4%	15.8%
proprietary solution			X							X		

- ΔFI – relative error between predicted and experimental fatigue limit
  - ΔFI =0 ideal
  - ΔFI >0 conservative
  - ΔFI <0 non-conservative

FatLim Database

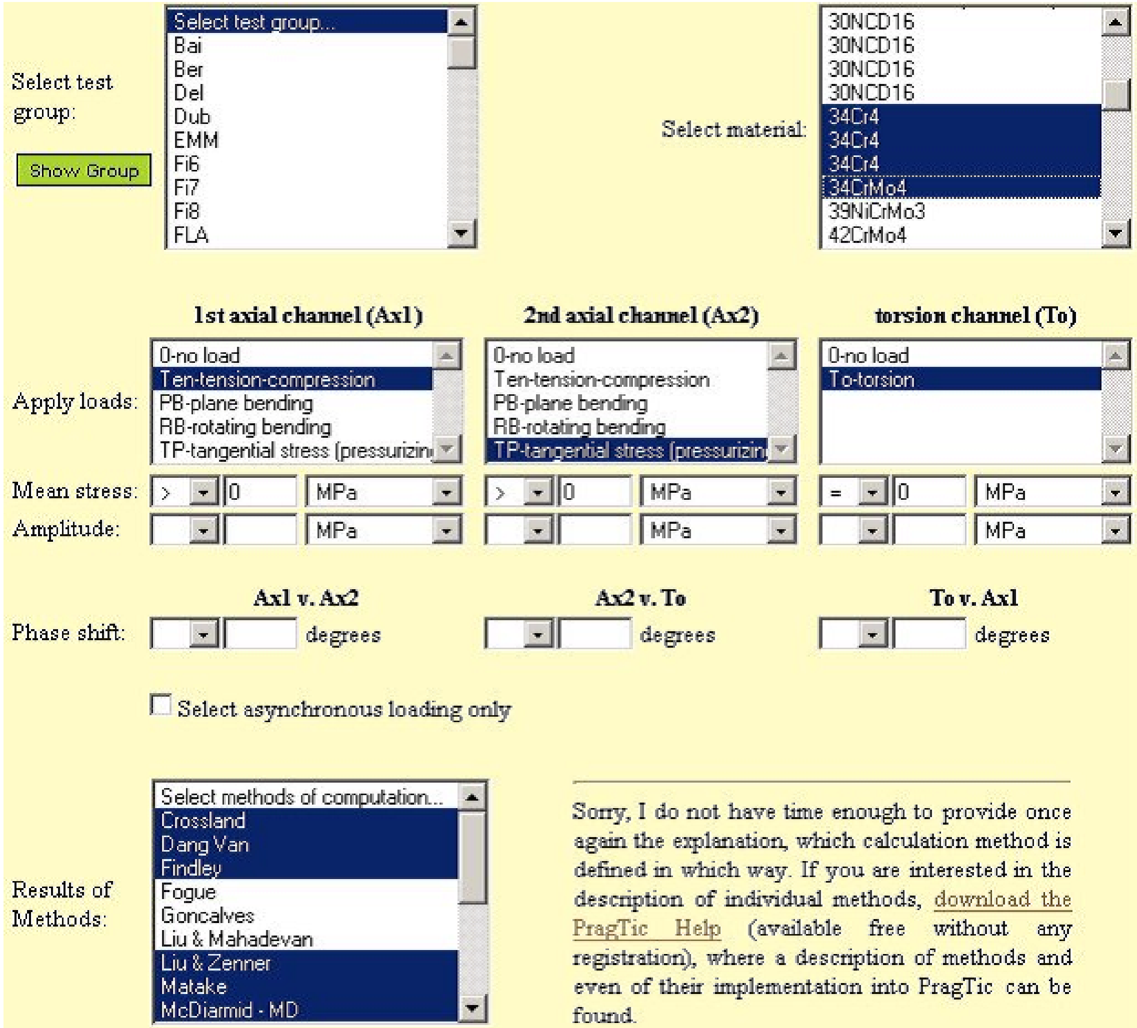
Presently:

- 451 experiments
- 18 calculation methods

Statistic evaluation of the fatigue index errors for the selected conditions available

Beware: Had to be replaced by newer and better checked FADOFF database

www.pragtic.com/experiments.php



Dang Van criterion

$$a_{DV} \cdot C_a + b_{DV} \cdot \sigma_{H,\max} \le f_{-1}$$

Critical plane criterion

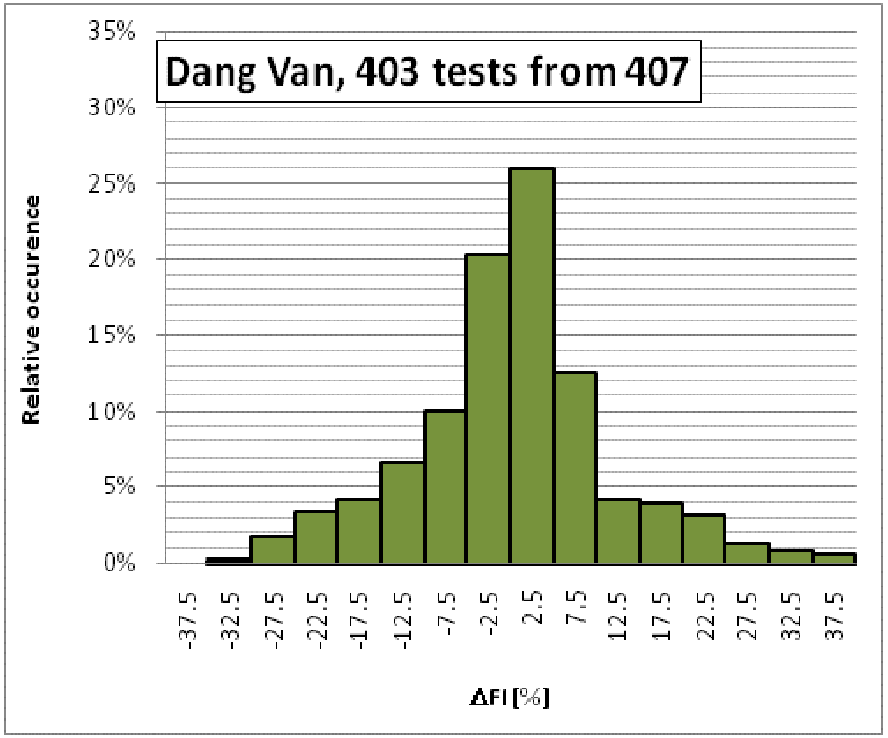
The most often used representative of multiaxial criteria

Use of maximum hydrostatic stress does not seem to give acceptable results

Conservative: MS, Ax+Ax

Non-conservative:

- **nMS, OP**
- **MS,To**



average: -0.1%

range: 92.9%

standard deviation: 12.2%

Weakness of Dang Van method

Hard to believe: multiaxial fatigue

Mean values of ΔFI in individual groups (tests)	CRO	DV
All (407)	*−8.0*	−0.1
nMS (171)	−3.1	−0.6
nMS,OP (40)	*−11.5*	*−7.9*
nMS,IP (131)	−0.5	1.7

relative difference between predicted and experimental fatigue limit

no mean stress, out-of-phase loading

no mean stress, in-phase loading

- There is a significant difference in mean prediction values depending on the phase shift of individual load channels

Manson-McKnight

So simple, that it can be computed in MS Excel

- !The cycle has to be detected a priori!

Amplitude and mean value of each stress component is evaluated:

$$\sigma_a = \sqrt{\frac{1}{2}\left[\left(\sigma_{x,a} - \sigma_{y,a}\right)^2 + \left(\sigma_{y,a} - \sigma_{z,a}\right)^2 + \left(\sigma_{z,a} - \sigma_{x,a}\right)^2 + 6\left(\tau_{xy,a}^2 + \tau_{yz,a}^2 + \tau_{zx,a}^2\right)\right]}$$

$$\sigma_m^* = sign\left[I_{1,d}\right] \cdot \sqrt{\frac{1}{2}\left[\left(\sigma_{x,m} - \sigma_{y,m}\right)^2 + \left(\sigma_{y,m} - \sigma_{z,m}\right)^2 + \left(\sigma_{z,m} - \sigma_{x,m}\right)^2 + 6\left(\tau_{xy,m}^2 + \tau_{yz,m}^2 + \tau_{zx,m}^2\right)\right]}$$

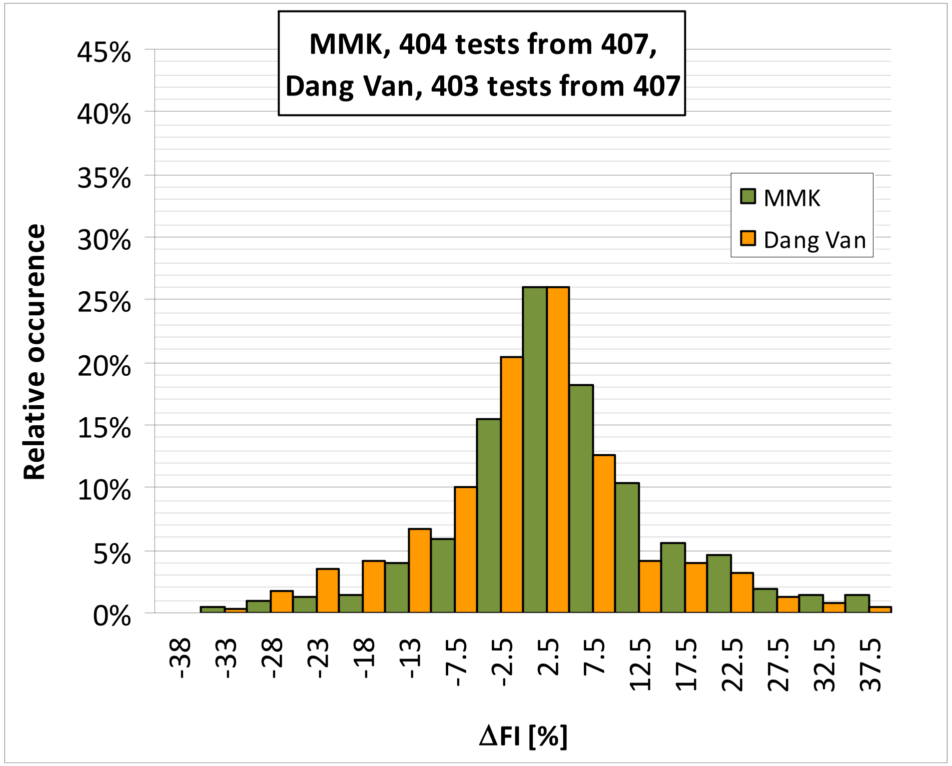
The mean equivalent value is signed according to the stress tensor invariant with biggest magnitude

Manson-McKnight - Results

Not that bad as by signed von Mises

Many evaluated classes of experiments with conservative mean value (To; nMS-OP; Ax+To; brittle materials)

- Ax+Ax with phase shift – dangerously non-conservative (mean value ΔFI=-17.2% !)
- In evaluation of individual classes is the Dang Van method better, but fails in mean stress effect



MMK versus Dang Van

DV – Dang Van critical plane method (1974)

MMKF –Manson-McKnight according to Filippini (2010)

MMK not to be used for

- brittle materials
- MS,Ax+Ax, PS<>0
- out-of-phase loading

MMK useful for pressure vessels

- MS,Ax+Ax,noPS

The difference is not big overall!

ΔFI in individual groups (tests)	Mean values			Range			Standard deviation		
	DV	MMKF	Diff	DV	MMKF	Diff	DV	MMKF	Diff
All (407)	0	1	1	93	106	13	12	12	-1
nMS (171)	-1	5	5	53	71	18	8	9	1
nMS,OP (40)	-8	8	16	53	70	18	12	14	2
nMS,IP (131)	2	4	2	22	45	22	4	7	3
MS (236)	0	-1	-2	93	82	-11	15	12	-2
MS,Ax (41)	-1	0	1	64	62	-2	13	11	-2
MS,To (18)	-16	-5	11	46	65	19	12	16	4
MS,Ax+Ax (36)	12	-11	-23	57	43	-14	13	12	-1
MS,Ax+Ax,noPS (18)	15	-3	-18	43	22	-20	12	8	-4
MS,Ax+Ax, PS<>0 (18)	9	-19	-28	57	40	-18	14	10	-4
Ax+To (285)	-2	3	5	63	96	33	10	10	0
MS,Ax+To (114)	-3	0	3	61	66	5	12	10	-1
MS-Ax, Ax+To (52)	-1	6	7	61	43	-18	12	9	-4
MS-To, Ax+To (31)	-10	-6	3	41	41	1	9	8	-1
ductile (352)	0	0	0	93	82	-11	13	11	-2
ductile,nMS (118)	-1	4	4	43	30	-12	8	5	-3
ductile,nMS,IP (86)	3	3	1	18	18	1	4	5	1
brittle (37)	-1	11	12	42	75	33	7	16	9
brittle, nMS(35)	-1	12	13	42	70	27	7	16	9
brittle,nMS,IP (29)	-2	8	10	16	44	28	3	11	8
extra-ductile (18)	2	0	-2	19	18	-1	5	4	-1
extra-ductile,nMS,IP (16)	3	-1	-4	12	11	-2	3	3	0

New results – ΔFI in fatigue limit estimate

New MMP model – equivalent stress, no PragTic, just pure Excel solution!
Published in International Journal of Fatigue, 2017

Abbrev.	Description		average				range				standard deviation			
		Items	MMP	MMKF	PCRN	DV	MMP	MMKF	PCRN	DV	MMP	MMKF	PCRN	DV
All	All data items	307	-1.6%	-1.1%	1.6%	-2.4%	71.8%	142.0%	67.7%	104.8%	10.5%	15.0%	7.6%	16.7%
MS	some mean stress applied	183	-1.9%	-5.0%	1.8%	-4.3%	71.8%	134.3%	51.8%	104.8%	12.3%	17.0%	8.0%	19.7%
MS-	negative mean stress applied	22	5.2%	0.1%	4.4%	1.0%	69.5%	131.0%	38.6%	83.1%	18.8%	25.3%	11.2%	20.4%
nMS-	Without cases with negative mean stress	285	-2.1%	-1.2%	1.4%	-2.7%	69.7%	87.0%	67.7%	104.8%	9.4%	13.9%	7.2%	16.3%
nMS	no mean stress applied	122	-1.0%	4.8%	1.6%	0.6%	60.5%	68.8%	67.7%	81.4%	7.0%	9.1%	7.0%	10.2%
brittle	brittle material with kappa < 1.3	34	-4.3%	9.7%	2.9%	2.5%	52.7%	42.1%	38.6%	96.9%	9.6%	11.2%	8.2%	19.6%
brit,nMS	brittle material without any mean stress applied	19	-5.8%	12.0%	0.4%	0.3%	24.2%	32.5%	21.1%	41.2%	6.2%	8.8%	4.9%	9.5%
ex-duct	extra-ductile material with kappa > sqrt(3)	43	-7.7%	-13.3%	-4.1%	-16.8%	46.1%	57.1%	45.9%	60.4%	11.6%	15.0%	8.5%	15.7%
ed,nMS	extra-ductile material without any mean stress applied	10	-5.4%	-7.2%	-4.6%	-5.8%	39.4%	38.1%	45.9%	49.8%	12.7%	12.6%	14.1%	15.8%
ed,2A	extra-ductile material with multichannel loading	18	-10.9%	-13.6%	-5.8%	-15.2%	46.1%	52.5%	45.9%	54.4%	13.8%	14.3%	10.9%	17.5%
AT	interaction of axial and torsion loading	203	-3.1%	2.1%	1.8%	-3.0%	67.2%	83.2%	67.7%	86.0%	8.6%	12.6%	7.9%	13.9%
P,AT	proportional interaction of axial and torsion loading	106	-2.5%	2.3%	1.3%	0.4%	54.3%	67.1%	55.5%	85.3%	7.2%	10.7%	6.4%	11.2%
P,AT,MS	proportional interaction of axial and torsion loading - with mean stresses	11	-14.3%	-18.9%	-6.4%	-23.7%	25.4%	26.6%	25.4%	37.2%	7.6%	10.1%	8.0%	12.6%
NP,AT	non-proportional interaction of axial and torsion loading	96	-4.1%	1.5%	2.2%	-7.0%	49.9%	75.0%	60.8%	76.3%	9.1%	13.9%	9.0%	15.4%
NP,AT,MS	non-proportional interaction of axial and torsion loading with MS	70	-4.9%	0.5%	3.5%	-6.1%	49.9%	75.0%	40.8%	76.3%	9.3%	14.8%	8.5%	16.1%
NP,AT,nMS	non-proportional interaction of axial and torsion loading without MS	28	-2.0%	3.8%	-1.3%	-9.2%	39.4%	51.6%	45.9%	58.2%	8.1%	10.4%	9.2%	13.2%
NP,IP,AT	non-proportional in-phase interaction of axial and torsion loading	12	-4.3%	2.5%	5.3%	-0.4%	28.0%	34.7%	15.9%	32.9%	7.2%	8.4%	4.1%	9.0%
NP,OP,AT	non-proportional out-of-phase interaction of axial and torsion loading	51	-5.1%	1.0%	-0.9%	-12.8%	46.1%	56.5%	48.3%	62.3%	9.5%	11.3%	8.6%	13.8%
2-3A	interaction of two axial and potentially also torsion load channels	39	6.4%	-9.3%	2.1%	12.1%	58.4%	49.4%	29.4%	59.4%	16.5%	10.1%	6.3%	15.1%
2A	interaction of two axial load channels	18	10.0%	-12.7%	0.8%	7.5%	56.2%	37.1%	17.7%	59.4%	17.7%	9.9%	5.2%	17.8%
IP	in-phase loading (load path intersecting the <0,0> point)	104	-2.7%	1.8%	1.1%	0.4%	54.3%	67.1%	55.5%	85.3%	7.1%	10.8%	6.4%	11.6%
IP,MS	in-phase loading (load path intersecting the <0,0> point)	12	-13.7%	-18.6%	-5.9%	-18.9%	25.4%	26.6%	25.4%	74.5%	7.5%	9.7%	7.8%	20.0%
IP,nMS	in-phase loading without any mean stress applied	92	-1.3%	4.4%	2.0%	3.0%	45.1%	59.5%	55.5%	65.6%	5.7%	7.6%	5.5%	6.7%
IP,nMS,s	in-phase loading without any mean stress applied and prevalent axial stress	49	-0.5%	1.7%	1.5%	2.2%	31.5%	36.3%	30.5%	35.5%	5.0%	5.3%	4.4%	5.5%
IP,nMS,t	in-phase loading without any mean stress applied and prevalent torsion channel	43	-2.3%	7.1%	2.4%	3.6%	37.5%	52.8%	46.8%	55.6%	6.3%	8.7%	6.5%	7.8%
MS,Ax	axial loading only (with mean stress involved)	42	-1.3%	-2.8%	1.2%	-5.8%	54.7%	134.3%	38.0%	104.8%	10.1%	20.2%	6.5%	21.5%
MS,To	torsion loading only (with mean stress involved)	21	-2.9%	-14.9%	-0.8%	-18.4%	34.8%	72.6%	42.0%	39.9%	7.7%	18.0%	8.4%	13.1%
OP	out-of-phase loading	77	-2.7%	-2.6%	0.2%	-3.3%	67.2%	74.7%	56.5%	78.6%	11.4%	13.2%	8.0%	18.7%
OP,nMS	out-of-phase loading without any mean stress applied	28	-0.9%	5.2%	-0.7%	-8.8%	60.5%	68.8%	56.5%	58.2%	10.5%	12.6%	10.3%	13.8%
OP,MS	out-of-phase loading with some mean stress applied	49	-3.7%	-7.0%	0.8%	-0.2%	66.6%	48.3%	28.2%	78.6%	11.7%	11.4%	6.2%	20.3%
OP,MS,AT	out-of-phase loading with some mean stress applied, axial-torsional load cases only	24	-8.4%	-2.3%	0.0%	-16.3%	38.2%	47.4%	28.2%	45.6%	9.8%	11.1%	7.8%	13.8%
OP,MS,2A	out-of-phase loading with some mean stress applied, bi-axial load cases only	9	3.3%	-17.3%	2.1%	13.2%	53.2%	37.1%	14.4%	39.4%	14.8%	11.0%	4.2%	14.6%
OP,MS,3A	out-of-phase loading with some mean stress applied and triaxial load states	16	-0.6%	-8.3%	1.2%	16.4%	31.8%	26.9%	15.6%	37.1%	9.0%	7.2%	4.0%	9.7%

Crossland criterion

$$a_C \cdot \left(\sqrt{J_2}\right)_a + b_C \cdot \sigma_{H,\max} \le f_{-1}$$

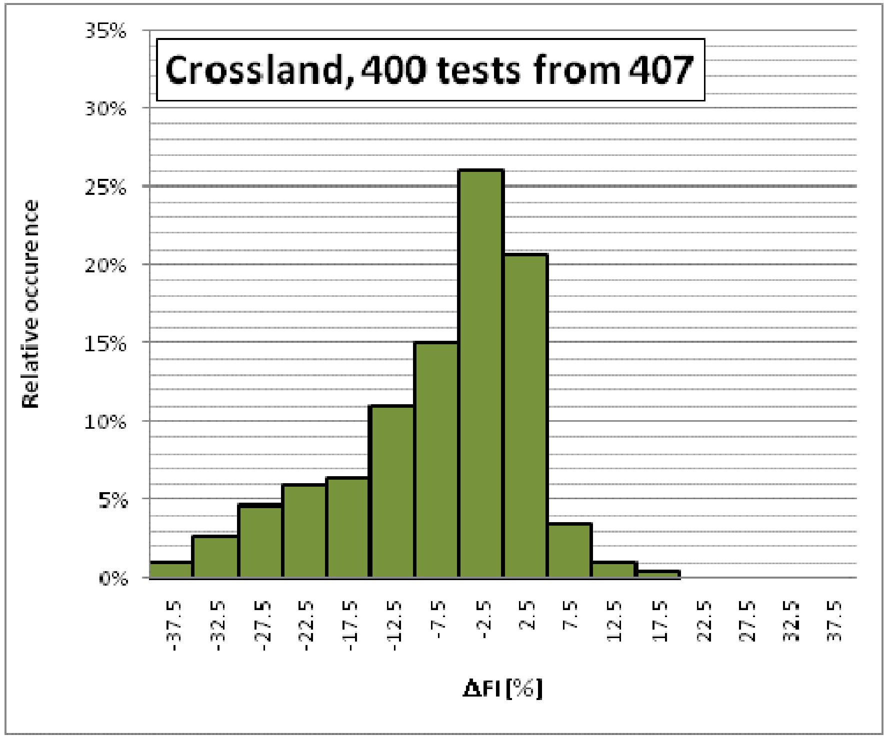
IDS type, simple calculation

Fascinating range in comparison to the Sines' version

Significantly non-conservative mean value, high standard deviation

Non-conservative:

- **nMS,OP**
- **MS**



average: -8.0%

range: 64.9%

standard deviation: 11.4%

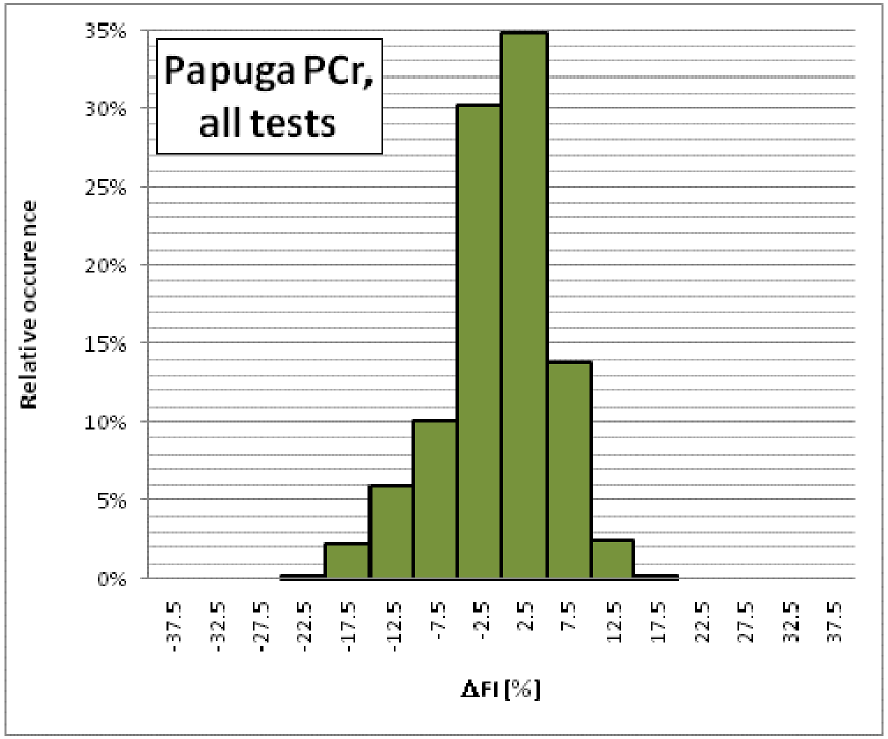
Papuga PCr criterion

$$\sqrt{a_c \cdot C_a^2 + b_c \cdot \left(N_a + \frac{t_{-1}}{f_0} N_m \right)} \leq f_{-1}$$

$$\kappa < \sqrt{\frac{4}{3}} \cong 1.155 \;:\; a_c = \frac{\kappa^2}{2} + \frac{\sqrt{\kappa^4 - \kappa^2}}{2}, \qquad b_c = f_{-1}$$

$$\kappa \geq \sqrt{\frac{4}{3}} \cong 1.155 \;:\; a_c = \left(\frac{4\kappa^2}{4+\kappa^2} \right)^2, \; b_c = \frac{8 f_{-1} \kappa^2 \left(4-\kappa^2\right)}{\left(4+\kappa^2\right)^2}$$

- critical approach of MD type
- different definition of a and b parameters based on value is the output of the mathematical analysis
- Published in Int J Fat 1/2008
- Non-conservative for MS,To
- Available in PragTic fatigue solver



average: -0.5%

range: 37.4%

standard deviation: 6.1%

PCr vs. PCrN (available now in PragTic)

The change affects (MS,To) group in the mean value and shifts it to the right position.

(MS,Ax) and (MS,Ax+Ax) and derived groups are also improved as regards their scatter. They are uniformly shifted to over-conservative mean values.

The effect on (MS, Ax+To) group is not so pronounced.

Unpublished yet

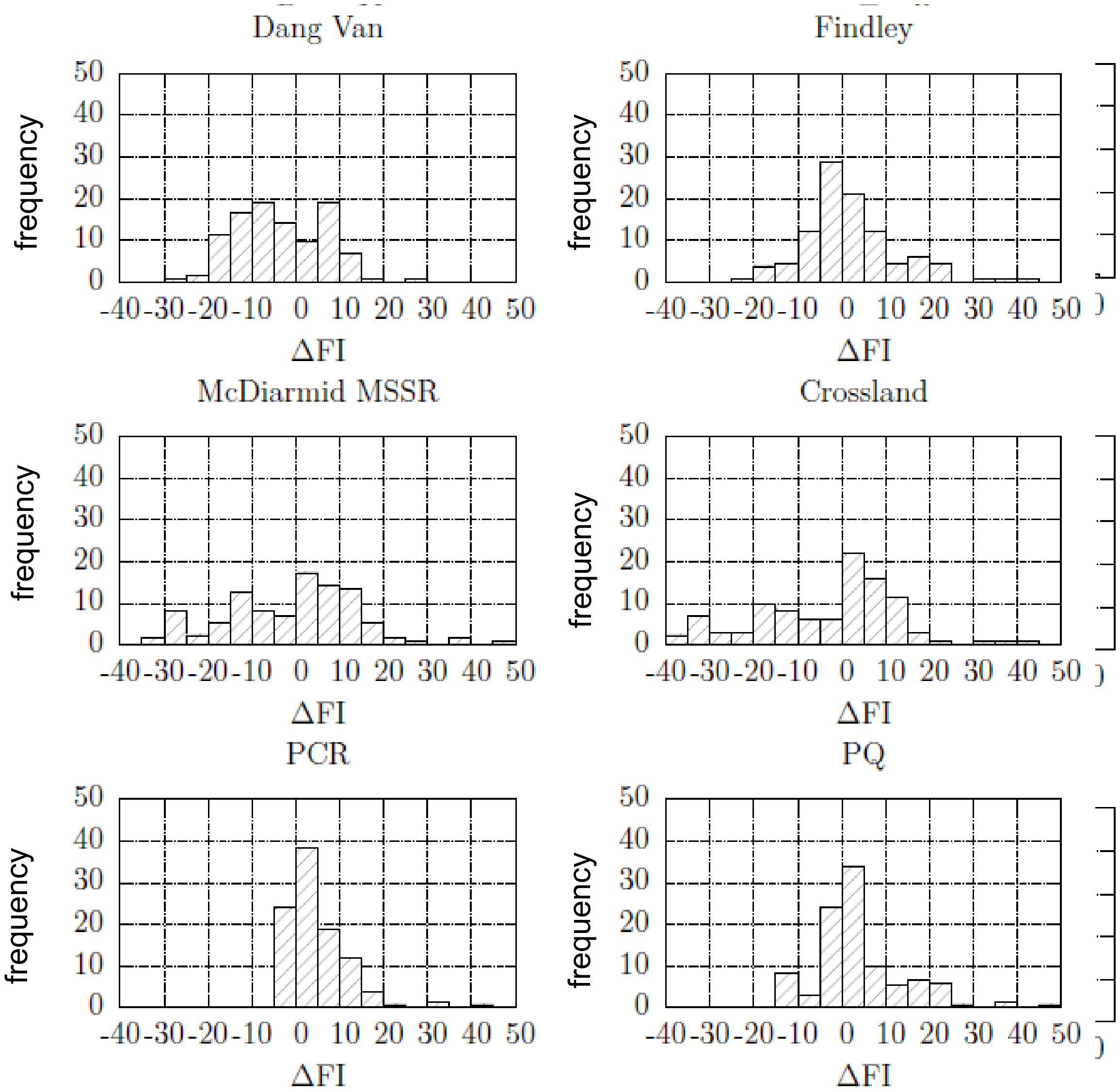
ΔFI in individual groups (tests)	Mean values			Range			Standard		
	PCr	PCrN	Diff	PCr	PCrN	Diff	PCr	PCrN	Diff
All (407)	-1	1	2	37	31	-6	6	5	-1
nMS (171)	1	1	0	23	23	0	4	4	0
nMS,OP (40)	0	0	0	23	23	0	6	6	0
nMS,IP (131)	1	1	0	17	16	0	3	3	0
MS (236)	-1	2	3	37	31	-6	7	6	-2
MS,Ax (41)	-2	2	4	33	24	-9	7	5	-2
MS,To (18)	-8	-2	7	17	16	0	5	4	-1
MS,Ax+Ax (36)	-4	4	8	32	20	-12	7	5	-2
MS,Ax+Ax,noPS (18)	-4	4	8	25	19	-5	8	6	-2
MS,Ax+Ax, PS<>0 (18)	-4	4	8	27	15	-11	7	4	-3
Ax+To (285)	1	1	1	36	31	-4	5	5	0
MS,Ax+To (114)	1	2	1	36	31	-4	6	6	0
MS-Ax, Ax+To (52)	0	3	3	28	31	3	6	6	0
MS-To, Ax+To (31)	3	3	0	24	19	-5	5	5	0
ductile (352)	-1	2	2	37	31	-6	6	5	-1
ductile,nMS (118)	1	1	0	22	22	0	4	4	0
ductile,nMS,IP (86)	2	2	0	14	14	0	3	3	0
brittle (37)	0	-1	0	17	17	0	4	4	0
brittle, nMS(35)	-1	-1	0	17	17	0	4	4	0
brittle,nMS,IP (29)	-2	-2	0	9	9	0	2	2	0
extra-ductile (18)	2	2	0	12	12	0	3	3	0
extra-ductile,nMS,IP (16)	3	3	0	12	12	0	3	3	0

PCr in finite life estimation

Low-cycle fatigue (<50000 cycles)

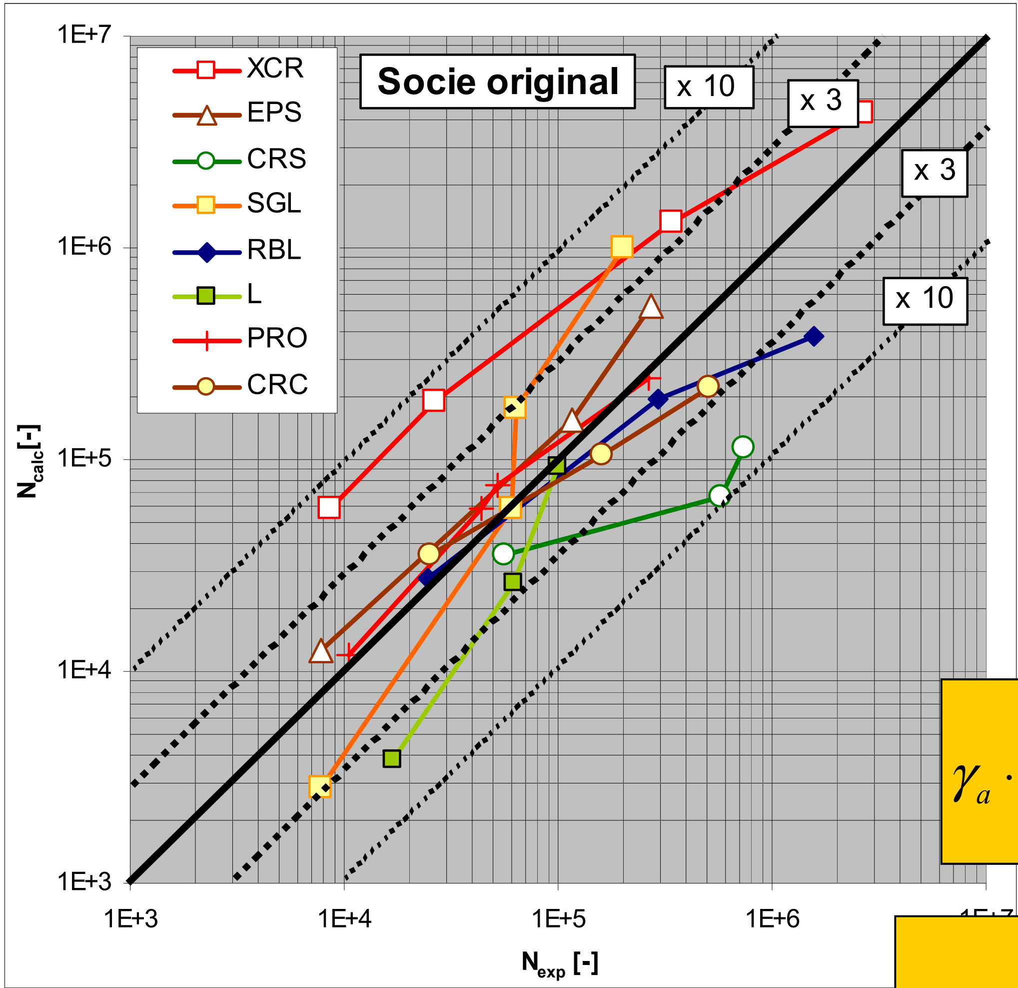
- commercially used methods have bigger scatter there than PCr
- Dang Van method shows another peak around ΔFI=–8%

Only unnotched specimens evaluated



Multiaxial Methods for Limited Lifetime

Socie et al.



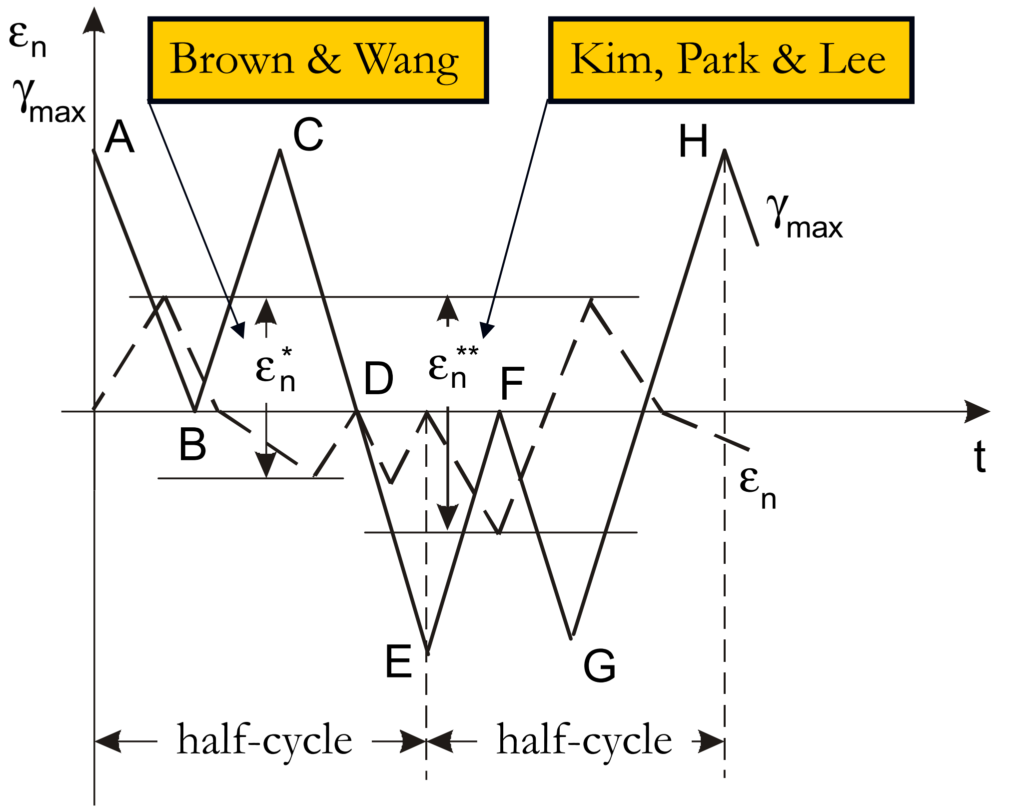
- Combination of two calculation methods – each related to different mode of crack initiation
- Only the shear or tensile variant used in some cases

$$\gamma_a \cdot \left(1 + k_1 \cdot \frac{\sigma_{n,max}}{\sigma_y}\right) = \frac{\tau_f'}{G} \cdot (2N)^b + \gamma_f' \cdot (2N)^c$$

$$\sigma_{n,\max} \cdot \varepsilon_{na} = \frac{\sigma_f'^2}{E} \cdot (2N)^{2b} + \sigma_f' \cdot \varepsilon_f' \cdot (2N)^{b+c}$$

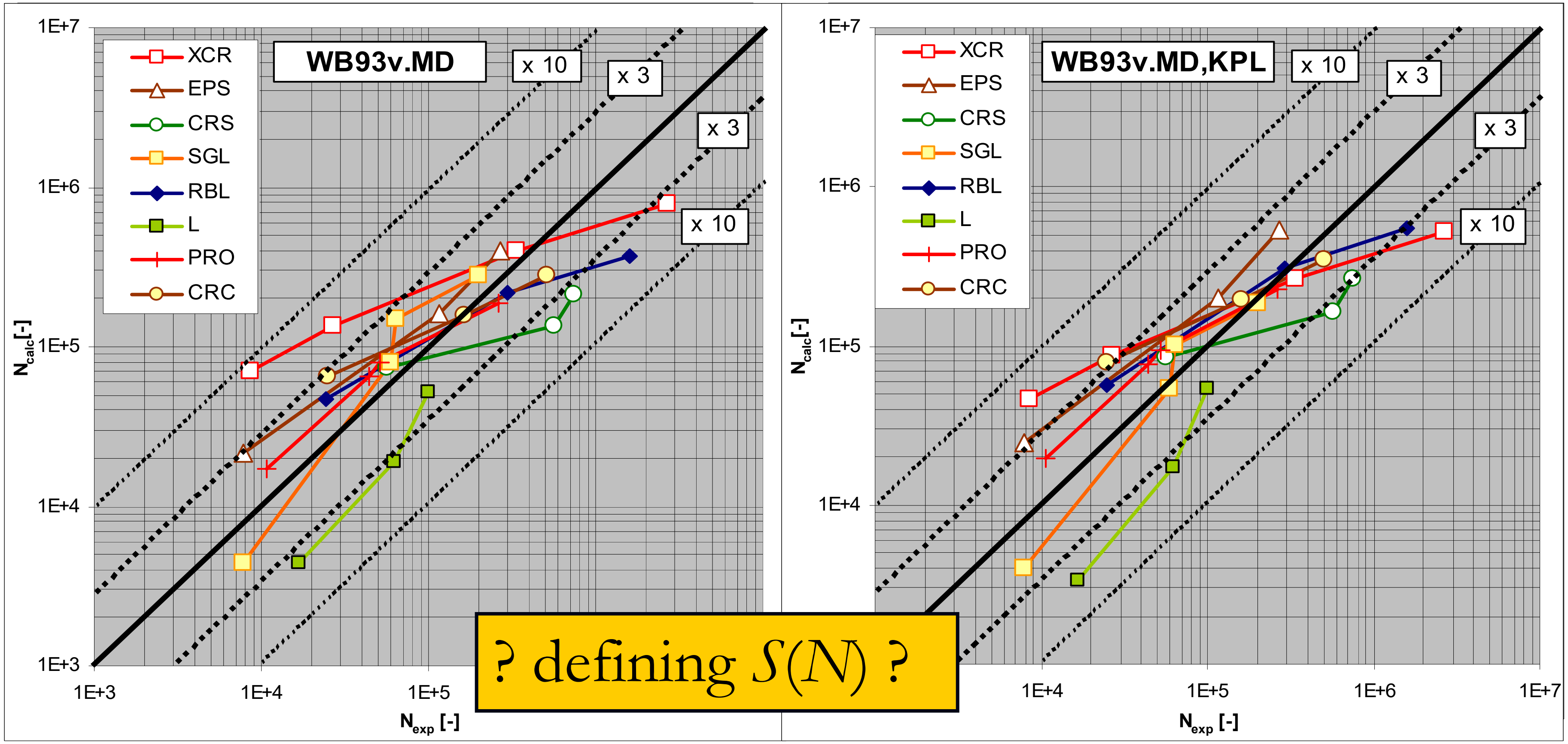
Brown – Wang v. '93

- Shear strain decisive
- The normal strain range present in the shear strain cycle is derived
- Originally MSSR criterion
- Allowed MD in PragTic
- Mean stress effect enabled (modification of the Basquin formula)
- Material parameter S set from t_{-1}, f_{-1} values (but authors prefer setting by fit to experiments)



$$\frac{\Delta\gamma}{2} = \frac{\Delta\gamma_{max}}{2} + S \cdot \varepsilon_n^*$$

Results of W&B Criterion



- **MSSR versions**
  - non-conservative for the *S* derived from fatigue limits
  - large scatter for *S* fitted to mean *LLR* 0
- **MD versions**
  - better scatter (KPL method wins)
  - obvious trend from conservative prediction in HCF to non-conservative prediction in LCF visible even here