In[270]:=
```
ClearAll["Global`*"]
```

Plasticita - prutová soustava

Toto je pouze řešení v matematickém sw, výklad k němu proběhl na cvičení 6.3. 2020.
T.Hruš

In[271]:=
$$\text{cisla} = \{a \to 3,\ b \to 4,\ c \to 8.5,\ S \to 20 \times 10^{-6},\ \mathbb{E} \to 2.1 \times 10^{11},\ \sigma k \to 500 \times 10^{6}\};$$

In[272]:=
$$\text{delky} = \{l1 \to b,\ l2 \to \sqrt{a^2 + b^2},\ l3 \to \sqrt{a^2 + c^2}\};$$
$$\text{trig} = \left\{\text{Sin}[\beta] \to \frac{b}{l2},\ \text{Cos}[\beta] \to \frac{a}{l2},\ \text{Sin}[\gamma] \to \frac{c}{l3},\ \text{Cos}[\gamma] \to \frac{a}{l3}\right\};$$

In[274]:=
$$\begin{aligned}\text{rr1} = \{&\\ &\sigma 3\, S\, \text{Cos}[\gamma] + Rx + \sigma 2\, S\, \text{Cos}[\beta] == 0,\\ &\sigma 3\, S\, \text{Sin}[\gamma] + Ry + \sigma 2\, S\, \text{Sin}[\beta] + \sigma 1\, S - F == 0,\\ &\sigma 3\, S\, \text{Sin}[\gamma]\, a - \sigma 2\, S\, \text{Sin}[\beta]\, a - \sigma 1\, S\, 2\, a + F\, a == 0\\ \};&\end{aligned}$$

In[275]:=
$$\begin{aligned}\text{fr1} = \{&\\ &\Delta l1 == \frac{\sigma 1}{\mathbb{E}}\, l1,\\ &\Delta l2 == \frac{\sigma 2}{\mathbb{E}}\, l2,\\ &\Delta l3 == \frac{\sigma 3}{\mathbb{E}}\, l3\\ \};&\end{aligned}$$

In[276]:=
$$\begin{aligned}\text{dr1} = \{&\\ &\Delta l1 == 2\, a\, \varphi,\\ &\Delta l2 == a\, \varphi\, \text{Sin}[\beta],\\ &\Delta l3 == -a\, \varphi\, \text{Sin}[\gamma]\\ \};&\end{aligned}$$

In[277]:=
```
rce1 = Join[rr1, dr1, fr1]
```

Out[277]=
$$\{Rx + S\, \sigma 2\, \text{Cos}[\beta] + S\, \sigma 3\, \text{Cos}[\gamma] == 0,\ -F + Ry + S\, \sigma 1 + S\, \sigma 2\, \text{Sin}[\beta] + S\, \sigma 3\, \text{Sin}[\gamma] == 0,\ a\, F - 2\, a\, S\, \sigma 1 - a\, S\, \sigma 2\, \text{Sin}[\beta] + a\, S\, \sigma 3\, \text{Sin}[\gamma] == 0,\ \Delta l1 == 2\, a\, \varphi,\ \Delta l2 == a\, \varphi\, \text{Sin}[\beta],\ \Delta l3 == -a\, \varphi\, \text{Sin}[\gamma],\ \Delta l1 == \frac{l1\, \sigma 1}{\mathbb{E}},\ \Delta l2 == \frac{l2\, \sigma 2}{\mathbb{E}},\ \Delta l3 == \frac{l3\, \sigma 3}{\mathbb{E}}\}$$

In[278]:=
```
rce1 // TableForm
```

Out[278]//TableForm=

$$Rx + S\, \sigma 2\, \text{Cos}[\beta] + S\, \sigma 3\, \text{Cos}[\gamma] == 0$$
$$-F + Ry + S\, \sigma 1 + S\, \sigma 2\, \text{Sin}[\beta] + S\, \sigma 3\, \text{Sin}[\gamma] == 0$$
$$a\, F - 2\, a\, S\, \sigma 1 - a\, S\, \sigma 2\, \text{Sin}[\beta] + a\, S\, \sigma 3\, \text{Sin}[\gamma] == 0$$
$$\Delta l1 == 2\, a\, \varphi$$
$$\Delta l2 == a\, \varphi\, \text{Sin}[\beta]$$
$$\Delta l3 == -a\, \varphi\, \text{Sin}[\gamma]$$
$$\Delta l1 == \frac{l1\, \sigma 1}{\mathbb{E}}$$
$$\Delta l2 == \frac{l2\, \sigma 2}{\mathbb{E}}$$
$$\Delta l3 == \frac{l3\, \sigma 3}{\mathbb{E}}$$

In[279]:=
```
nezn1 = {Rx, Ry, σ1, σ2, σ3, Δl1, Δl2, Δl3, φ};
```

In[280]:=
```
Length[nezn] == Length[rce]
```

Out[280]= True

In[281]:=
```
res1 = Solve[rce1, nezn1][[1]]
```

Out[281]=

$$\left\{Rx \to -\frac{F\, l1\, (l3 \operatorname{Cos}[\beta] \operatorname{Sin}[\beta] - l2 \operatorname{Cos}[\gamma] \operatorname{Sin}[\gamma])}{4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2},\right.$$
$$Ry \to \frac{2\, F\, (l2\, l3 + l1\, l2 \operatorname{Sin}[\gamma]^2)}{4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2},\ \sigma1 \to \frac{2\, F\, l2\, l3}{S\, (4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2)},$$
$$\sigma2 \to \frac{F\, l1\, l3 \operatorname{Sin}[\beta]}{S\, (4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2)},$$
$$\sigma3 \to -\frac{F\, l1\, l2 \operatorname{Sin}[\gamma]}{S\, (4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2)},$$
$$\Delta l1 \to \frac{2\, F\, l1\, l2\, l3}{S\, \mathbb{E}\, (4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2)},$$
$$\Delta l2 \to \frac{F\, l1\, l2\, l3 \operatorname{Sin}[\beta]}{S\, \mathbb{E}\, (4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2)},$$
$$\Delta l3 \to -\frac{F\, l1\, l2\, l3 \operatorname{Sin}[\gamma]}{S\, \mathbb{E}\, (4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2)},$$
$$\left.\varphi \to \frac{F\, l1\, l2\, l3}{a\, S\, \mathbb{E}\, (4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2)}\right\}$$

In[282]:=
```
σ1 /. res1
```

Out[282]=

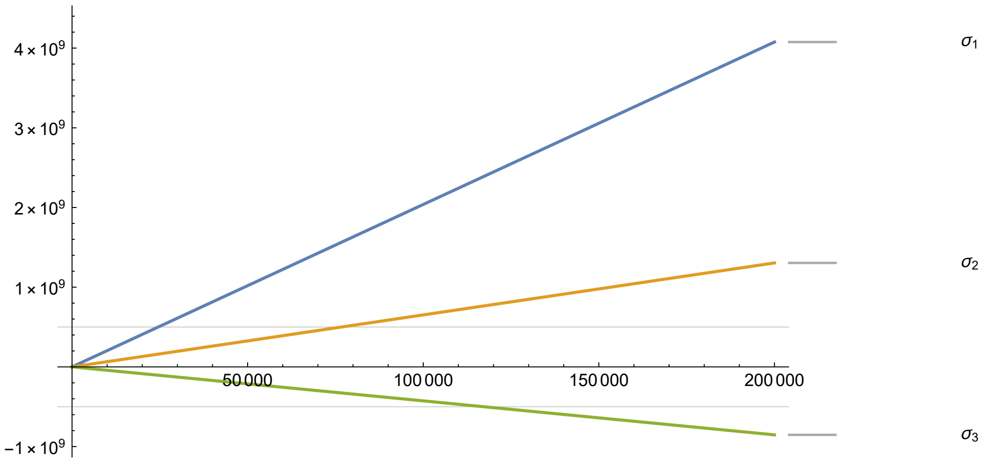
$$\frac{2\, F\, l2\, l3}{S\, (4\, l2\, l3 + l1\, l3 \operatorname{Sin}[\beta]^2 + l1\, l2 \operatorname{Sin}[\gamma]^2)}$$

In[283]:=
```
napeti1 = {σ1, σ2, σ3} /. res1 /. trig /. delky /. cisla
```

Out[283]= {20 380.7 F, 6521.82 F, -4264.27 F}

In[284]:=
```
Plot[napeti1, {F, 0, 200000},
 GridLines → {None, {-σk, +σk} /. cisla}, PlotLabels → {"σ₁", "σ₂", "σ₃"}]
```

Out[284]=

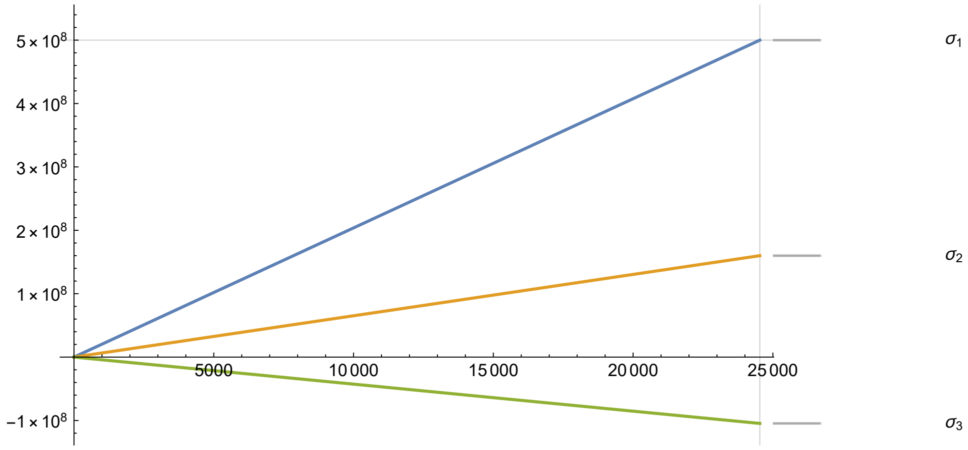


In[285]:=
```
FE = F /. Solve[(σ1 /. res1 /. trig /. delky /. cisla) == (σk /. cisla), F][[1]] // N
```

Out[285]= 24 533.

In[286]:=
```
Plot[napeti1, {F, 0, FE}, GridLines → {{FE}, {-σk, σk} /. cisla},
 PlotLabels → {"σ1", "σ2", "σ3"}, ImageSize → Large]
```

Out[286]=



In[287]:=
```
rr2 = rr1 /. {σ1 → σk}
```

Out[287]= $\{Rx + S\,\sigma2\,Cos[\beta] + S\,\sigma3\,Cos[\gamma] == 0,\ -F + Ry + S\,\sigma k + S\,\sigma2\,Sin[\beta] + S\,\sigma3\,Sin[\gamma] == 0,\ a\,F - 2\,a\,S\,\sigma k - a\,S\,\sigma2\,Sin[\beta] + a\,S\,\sigma3\,Sin[\gamma] == 0\}$

In[288]:=
```
dr2 = dr1
```

Out[288]= $\{\triangle l1 == 2\,a\,\varphi,\ \triangle l2 == a\,\varphi\,Sin[\beta],\ \triangle l3 == -a\,\varphi\,Sin[\gamma]\}$

In[289]:=
```
fr2 = fr1[[2 ;;]]
```

Out[289]= $\left\{\triangle l2 == \frac{l2\,\sigma2}{\mathbb{E}},\ \triangle l3 == \frac{l3\,\sigma3}{\mathbb{E}}\right\}$

In[290]:=
```
rce2 = Join[rr2, dr2, fr2]
```

Out[290]= $\left\{Rx + S\,\sigma2\,Cos[\beta] + S\,\sigma3\,Cos[\gamma] == 0,\ -F + Ry + S\,\sigma k + S\,\sigma2\,Sin[\beta] + S\,\sigma3\,Sin[\gamma] == 0,\ a\,F - 2\,a\,S\,\sigma k - a\,S\,\sigma2\,Sin[\beta] + a\,S\,\sigma3\,Sin[\gamma] == 0,\ \triangle l1 == 2\,a\,\varphi,\ \triangle l2 == a\,\varphi\,Sin[\beta],\ \triangle l3 == -a\,\varphi\,Sin[\gamma],\ \triangle l2 == \frac{l2\,\sigma2}{\mathbb{E}},\ \triangle l3 == \frac{l3\,\sigma3}{\mathbb{E}}\right\}$

In[291]:=
```
nezn2 = {Rx, Ry, σ2, σ3, Δl1, Δl2, Δl3, φ}
```

Out[291]= $\{Rx, Ry, \sigma2, \sigma3, \triangle l1, \triangle l2, \triangle l3, \varphi\}$

In[292]:=
```
Length[rce2] == Length[nezn2]
```

Out[292]= True

In[293]:=
```
res2 = Solve[rce2, nezn2][[1]]
```

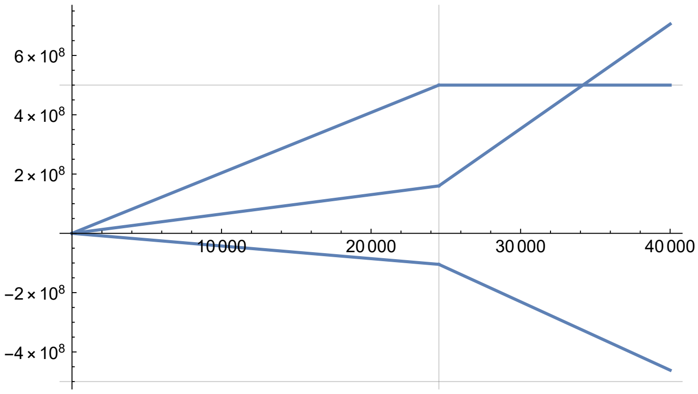
Out[293]=
$$\left\{Rx \to \frac{(-F + 2\,S\,\sigma k)\,(l3\,Cos[\beta]\,Sin[\beta] - l2\,Cos[\gamma]\,Sin[\gamma])}{l3\,Sin[\beta]^2 + l2\,Sin[\gamma]^2},\right.$$
$$Ry \to -\frac{-l3\,S\,\sigma k\,Sin[\beta]^2 - 2\,F\,l2\,Sin[\gamma]^2 + 3\,l2\,S\,\sigma k\,Sin[\gamma]^2}{l3\,Sin[\beta]^2 + l2\,Sin[\gamma]^2},$$
$$\sigma 2 \to -\frac{l3\,(-F + 2\,S\,\sigma k)\,Sin[\beta]}{S\,(l3\,Sin[\beta]^2 + l2\,Sin[\gamma]^2)},\ \sigma 3 \to \frac{l2\,(-F + 2\,S\,\sigma k)\,Sin[\gamma]}{S\,(l3\,Sin[\beta]^2 + l2\,Sin[\gamma]^2)},$$
$$\Delta l1 \to -\frac{2\,l2\,l3\,(-F + 2\,S\,\sigma k)}{S\,\mathbb{E}\,(l3\,Sin[\beta]^2 + l2\,Sin[\gamma]^2)},\ \Delta l2 \to -\frac{l2\,l3\,(-F + 2\,S\,\sigma k)\,Sin[\beta]}{S\,\mathbb{E}\,(l3\,Sin[\beta]^2 + l2\,Sin[\gamma]^2)},$$
$$\left.\Delta l3 \to \frac{l2\,l3\,(-F + 2\,S\,\sigma k)\,Sin[\gamma]}{S\,\mathbb{E}\,(l3\,Sin[\beta]^2 + l2\,Sin[\gamma]^2)},\ \varphi \to -\frac{l2\,l3\,(-F + 2\,S\,\sigma k)}{a\,S\,\mathbb{E}\,(l3\,Sin[\beta]^2 + l2\,Sin[\gamma]^2)}\right\}$$

In[294]:=
```
napeti2 = {σk, σ2, σ3} /. res2 /. trig /. delky /. cisla
```

Out[294]= {500 000 000, -35 296.5 (20 000 - F), 23 078.5 (20 000 - F)}

In[295]:=
```
Plot[{ napeti1  0 ≤ F < FE
       napeti2  FE ≤ F       , {F, 0, 40000}, GridLines → {{FE}, {-σk, σk} /. cisla}]
```

Out[295]=

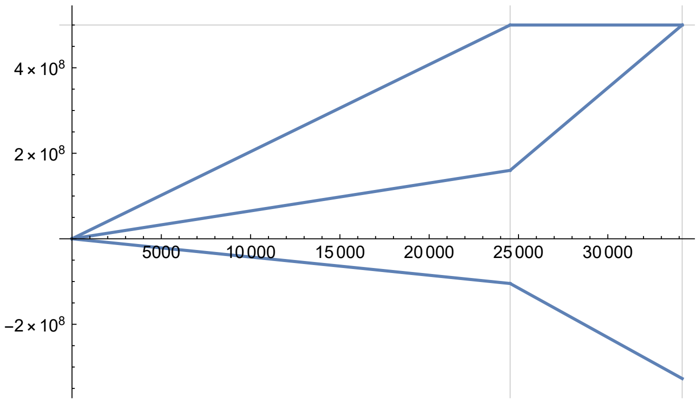


In[296]:=
```
FM1 = F /. Solve[(σ2 /. res2 /. trig /. delky /. cisla) == (σk /. cisla), F][[1]] // N
```

Out[296]= 34 165.7

In[297]:=
```
Plot[{ napeti1  0 ≤ F < FE
       napeti2  FE ≤ F       , {F, 0, FM1}, GridLines → {{FE, FM1}, {-σk, σk} /. cisla}]
```

Out[297]=



In[298]:=
```
rr3 = rr2 /. {σ2 → σk}
```

Out[298]=
$$\{Rx + S\,\sigma k\,Cos[\beta] + S\,\sigma 3\,Cos[\gamma] == 0,\ -F + Ry + S\,\sigma k + S\,\sigma k\,Sin[\beta] + S\,\sigma 3\,Sin[\gamma] == 0,$$
$$a\,F - 2\,a\,S\,\sigma k - a\,S\,\sigma k\,Sin[\beta] + a\,S\,\sigma 3\,Sin[\gamma] == 0\}$$

In[299]:=
```
nezn3 = {Rx, Ry, σ3};
```

In[300]:=
```
res3 = Solve[rr3, nezn3][[1]] /. {γ → ArcTan[c/a]} /. trig /. delky /. cisla // N
```

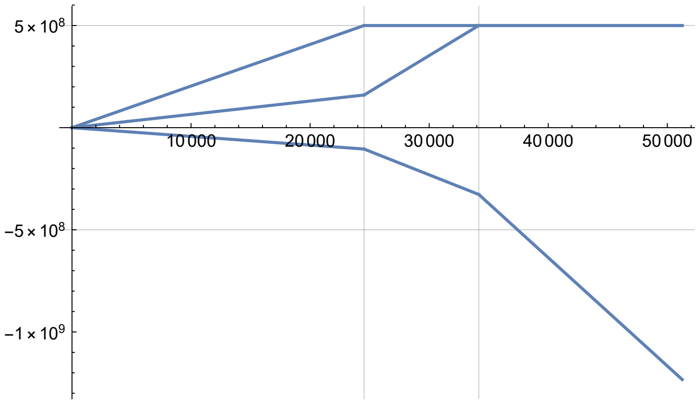
Out[300]= {Rx → -15 882.4 + 0.352941 F, Ry → -46 000. + 2. F, σ3 → -50 000. (-29 692.8 + 1.06046 F)}

In[301]:=
```
napeti3 = {σk, σk, σ3} /. res3 /. cisla
```

Out[301]= {500 000 000, 500 000 000, -50 000. (-29 692.8 + 1.06046 F)}

In[302]:=
```
Plot[{ napeti1   0 ≤ F < FE
       napeti2   FE ≤ F < FM1
       napeti3   FM1 ≤ F , {F, 0, 1.5 FM1},
 GridLines → {{FE, FM1}, {-σk, σk} /. cisla}]
```

Out[302]=

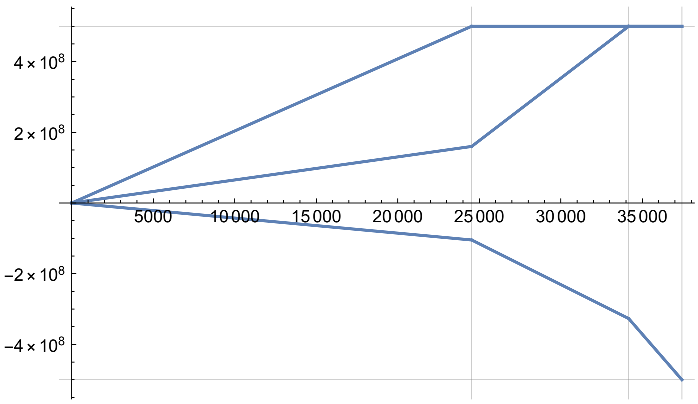


In[303]:=
```
FM2 = F /. Solve[
   (σ3 /. res3 /. {γ → ArcTan[c/a]} /. trig /. delky /. cisla // N) == -σk /. cisla, F][[1]]
```

Out[303]= 37 429.9

In[304]:=
```
Plot[{ napeti1   0 ≤ F < FE
       napeti2   FE ≤ F < FM1
       napeti3   FM1 ≤ F , {F, 0, FM2},
 GridLines → {{FE, FM1, FM2}, {-σk, σk} /. cisla}]
```

Out[304]=



Součinitel plast. rezervy

In[305]:=
```
FM2
───
FE
```

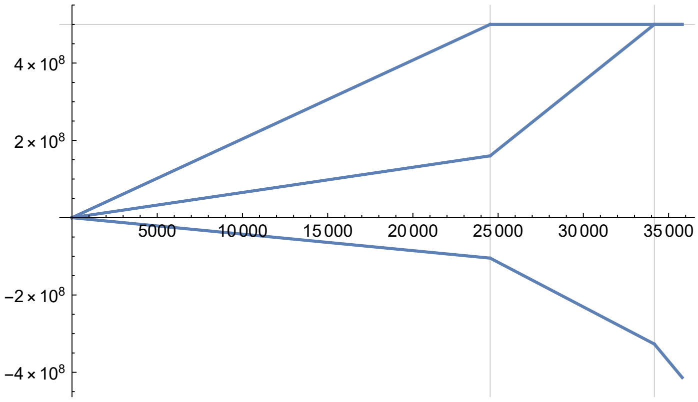
Out[305]= 1.52569

In[306]:= $Fp = \frac{FM1 + FM2}{2}$

Out[306]= 35 797.8

In[307]:= $\text{Plot}\left[\begin{cases} \text{napeti1} & 0 \le F < FE \\ \text{napeti2} & FE \le F < FM1 \\ \text{napeti3} & FM1 \le F \end{cases}, \{F, 0, Fp\},\right.$
$\left.\text{GridLines} \to \{\{FE, FM1, FM2\}, \{-\sigma k, \sigma k\} \mathbin{/.} \text{cisla}\}\right]$

Out[307]=



In[310]:= $k = \frac{\text{napeti1}}{F}$

Out[310]= {20 380.7, 6521.82, -4264.27}

In[313]:= σFp = {σk, σk, napeti3[[3]] /. F → Fp} /. cisla

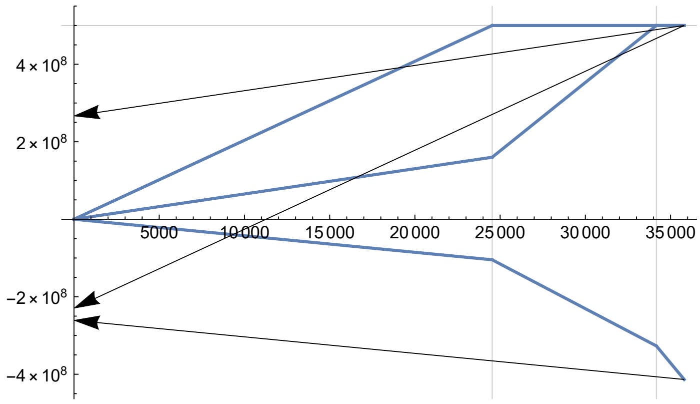
Out[313]= $\{500\,000\,000,\ 500\,000\,000,\ -4.13462 \times 10^8\}$

In[314]:= σzb = σFp - k Fp

Out[314]= $\{-2.29584 \times 10^8,\ 2.66533 \times 10^8,\ -2.6081 \times 10^8\}$

In[320]:=
```
Show[{
  Plot[ { napeti1   0 ≤ F < FE
          napeti2   FE ≤ F < FM1 , {F, 0, Fp},
          napeti3   FM1 ≤ F
   GridLines → {{FE, FM1, FM2}, {-σk, σk} /. cisla}]
  ,
  Graphics[{
    Arrow[{ {Fp, σFp[[1]]}, {0, σzb[[1]]}  }],
    Arrow[{ {Fp, σFp[[2]]}, {0, σzb[[2]]}  }],
    Arrow[{ {Fp, σFp[[3]]}, {0, σzb[[3]]}  }]
   }]
}]
```

Out[320]=



In[327]:=
```
(rr1 /. {σ1 → σzb[[1]], σ2 → σzb[[2]], σ3 → σzb[[3]], F → 0} /. trig /. delky /. cisla) //
  FullSimplify
```

Out[327]= {1462.34 + Rx == 0, Ry == 5245.98, False}