Kinematics - Introduction

Iva Petríková, doc. Ing.Ph.D.

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Kinematics – analysis of motion

- Motion of particles, rigid bodies, systems of RB in space
- Analysis from geometric point of view, neglecting the forces and torque that produce the motion
- Absolute space denotes: 3D space, homogenous, isotropic Euclidean (E_3) space
- Absolute time indicates constantly changing quantity
- Basic space (so called stationary) – system rigidly connected to the Earth. The body is in state of relative rest
- Units: 1m, 1s, 1 rad
- Displacement, speed, acceleration

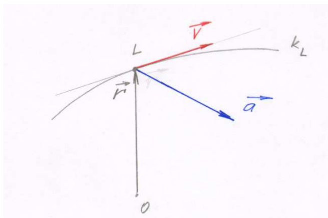
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Kinematics – analysis a synthesis

- Motion of a particle
- Motion of a rigid body
- Kinematics of mechanisms
- Kinematics of simultaneous motion
- Synthesis of mechanisms

Trajectory of motion, velocity and acceleration

- Radius vector **r** ($\vec{r}$)



- Velocity of a particle

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}}$$

- Acceleration of a particle

Trajectory of motion, velocity and acceleration

- Acceleration of a particle

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \dot{\vec{v}} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \ddot{\vec{r}}$$

Rectilinear motion

- The position of particle A at time t is determined by one coordinate x(t). Radius vector: $\vec{r} = x\vec{i}$
- The speed of that particle directed along OX is $v_x = \frac{dx}{dt} = \dot{x}(t)$ (1.1)
- The rate of change in speed of the particle is characterized by an acceleration (1.2)

$$a_x = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \ddot{x}(t)$$

- Vectors: $\vec{r} = x\,\vec{\imath}$ $\vec{v} = v\,\vec{\imath}$ $\vec{a} = a\,\vec{\imath}$
- Initial conditions: $x(t=0) = x_0$
  $v_x(t=0) = v_{x0}$
- Uniformly accelerated motion: initial velocity v_x is in agreement with positive direction of the OX axis

$$x = \frac{a_x t^2}{2} + v_{x0}t + x_0$$

- Uniformly decelerated motion: speed $v_x < 0$ and motion is opposite to the sence of acceleration

$$x = \frac{a_x t^2}{2} - v_{x0}t + x_0$$

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Calculation of velocity and acceleration

- Given: **x(t)** Determine: **v(t), a(t)**
- Differentiation of x(t) with respect to time - relations (1.1)

$$v(t) = \frac{dx}{dt}$$

- Differentiation of the velocity v(t) with respect to time - relations (1.1)

$$a(t) = \frac{dv}{dt}$$

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Calculation of acceleration and displacement

- Given: **v(t)** Determine: **x(t), a(t)**
- x(t):

  differential equation $v(t) = \frac{dx}{dt}$
- Solution of diff. eq.

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} v(t)\,dt \quad \Rightarrow \quad x - x_0 = \int_{t_0}^{t} v(t)\,dt$$

- Acceleration is given by the 1st derivative of the velocity with respect to time $a(t) = \frac{dv}{dt}$

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Calculation of acceleration and displacement

- Given: **a(t)** Determine: **v(t), x(t)**
- v(t):
  differential equation $a(t) = \frac{dv}{dt}$
- Solution of diff. eq.

$$\int_{v_0}^{v} dv = \int_{t_0}^{t} a(t)\,dt \quad \Rightarrow \quad v - v_0 = \int_{t_0}^{t} a(t)\,dt$$

- x(t), see slide N^0 8

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Calculation of acceleration and displacement

- Given: **a(x)** Determine: **v(x), x(t), t(x)**
- v(x):
  differential equation $a(x) = \frac{dv}{dt}\frac{dx}{dx} = v\frac{dv}{dx}$
- Solution of diff. eq.

$$v\,dv = a(x)\,dx \qquad \frac{v^2 - v_0^2}{2} = \int_{x_0}^{x} a(x)\,\mathrm{d}x$$

- x(t): $v(x) = \frac{dx}{dt}$
- Solution of diff. eq. $dt = \frac{dx}{v(x)}$ $\Rightarrow$ $t - t_0 = \int_{x_0}^{x} \frac{\mathrm{d}x}{v(x)}$

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ