Curvelinear motion of a particle

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Coordinates systems

- Rectangular coordinate system – x,y, (z)
- Normal and tangential c. – t,n, (b)
- Polar c. – φ, ρ Cylindrical c.- φ, ρ, z
- Spherical c.

Rectangular coordinates in space

- Parametric equations $x = x(t), y = y(t), z = z(t)$
- Radius vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$
- Velocity $\vec{v} = \dfrac{d\vec{r}}{dt} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$
- Acceleration $\vec{a} = \dfrac{d\vec{v}}{dt} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$
- Unit vectors $|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$

Rectangular coordinates in space

- Parametric equations $x = x(t), y = y(t), z = z(t)$
- Radius vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$
- Velocity components $v_x = \dot{x}\,;\, v_y = \dot{y}\,;\, v_z = \dot{z}$

$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

- Acceleration compon. $a_x = \dot{v}_x = \ddot{x}\,;\, a_y = \dot{v}_y = \ddot{y}\,;\, a_z = \dot{v}_z = \ddot{z}$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

- Magnitude of vel. vector $|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$
- Magnitude of acc. vector $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

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Natural coordinates

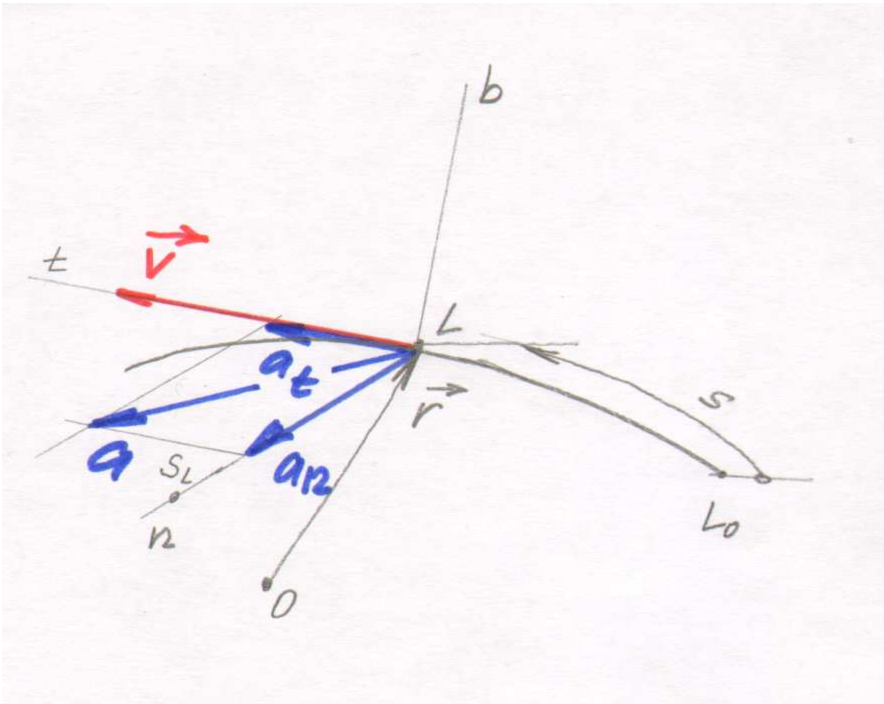
- Tangent axis, normal axis, binormal axis
- Parametric equation $s = s(t)$
- unit vectors – 3D $\vec{e}_t, \vec{e}_n, \vec{e}_b$, 2D $\vec{e}_t, \vec{e}_n$
- velocity $\vec{v} = v\vec{e}_t$
- acceleration $\vec{a} = a_t\vec{e}_t + a_n\vec{e}_n$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} = \dot{s}$$

$$a_t = \frac{\mathrm{d}v}{\mathrm{d}t} = \dot{v} = \ddot{s}$$

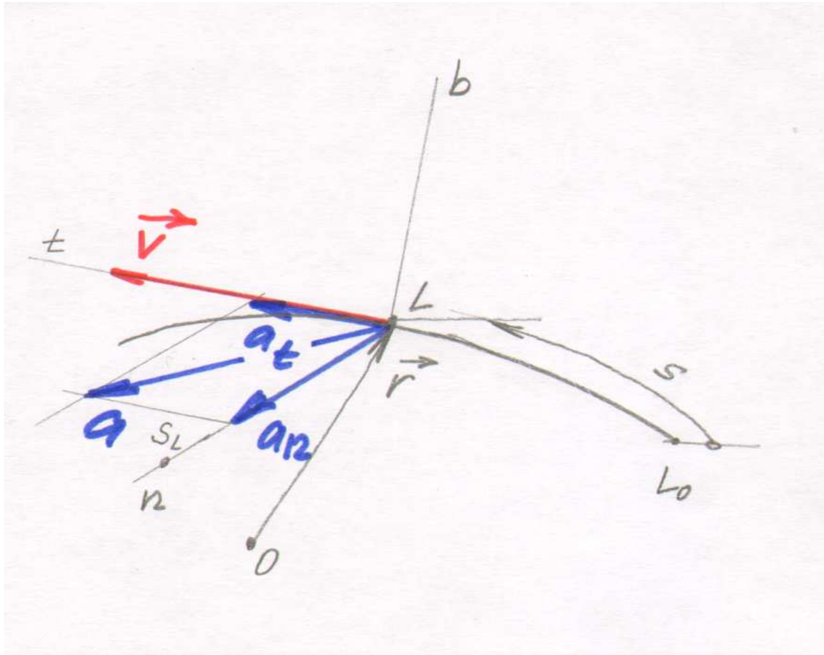
$$a_n = \frac{v^2}{\rho}$$

- ρ ... radius of curvature, $\rho = \overline{\mathrm{LS_L}}$



Natural coordinates

- velocity vector $\vec{v} = \dot{s}\vec{e}_t$
- acceleration vec. $\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{s}\vec{e}_t\right) = \ddot{s}\vec{e}_t + \dot{s}\frac{d\,\vec{e}_t}{dt} = \ddot{s}\vec{e}_t + \dot{s}^2\frac{d\vec{e}_t}{ds}$
- where $\frac{d\vec{e}_t}{dt} = \frac{d\vec{e}_t}{ds}\frac{ds}{dt}$ and $\frac{d\,\vec{e}_t}{ds} = \frac{\vec{e}_n}{\rho}$
- tangent component $a_t = \ddot{s} = \frac{d^2 s}{dt^2}$
- normal component $a_n = \frac{\dot{s}^2}{\rho}$

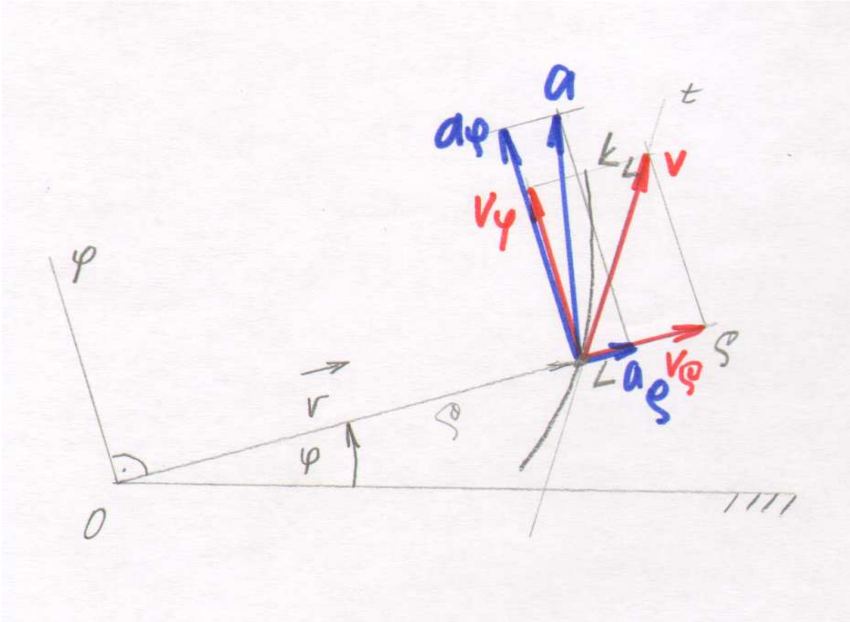


Polar coordinate system (2D)

- parametric equations $\rho = \rho(t), \varphi = \varphi(t)$
- unit vectors $\vec{e}_\rho, \vec{e}_\varphi$
- $\vec{r} = \rho e^{i\varphi}$ $\quad i = \sqrt{-1}$

$$\vec{v} = \dot{\vec{r}} = \dot{\rho} e^{i\varphi} + i\rho\dot{\varphi} e^{i\varphi}$$

- radial component $v_\rho = \frac{\mathrm{d}\rho}{\mathrm{d}t} = \dot{\rho}$
- transversal component $v_\varphi = \rho \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \rho\dot{\varphi}$



$$\vec{a} = \dot{\vec{v}} = \ddot{\rho} e^{i\varphi} + i\dot{\rho}\dot{\varphi} e^{i\varphi} + i\dot{\rho}\dot{\varphi} e^{i\varphi} + i\rho\ddot{\varphi} e^{i\varphi} + i^2 \rho\dot{\varphi}^2 e^{i\varphi}$$

$$a_\rho = \ddot{\rho} - \rho\dot{\varphi}^2 \qquad a_\varphi = \rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi}$$

Cyrindrical coordinate systems

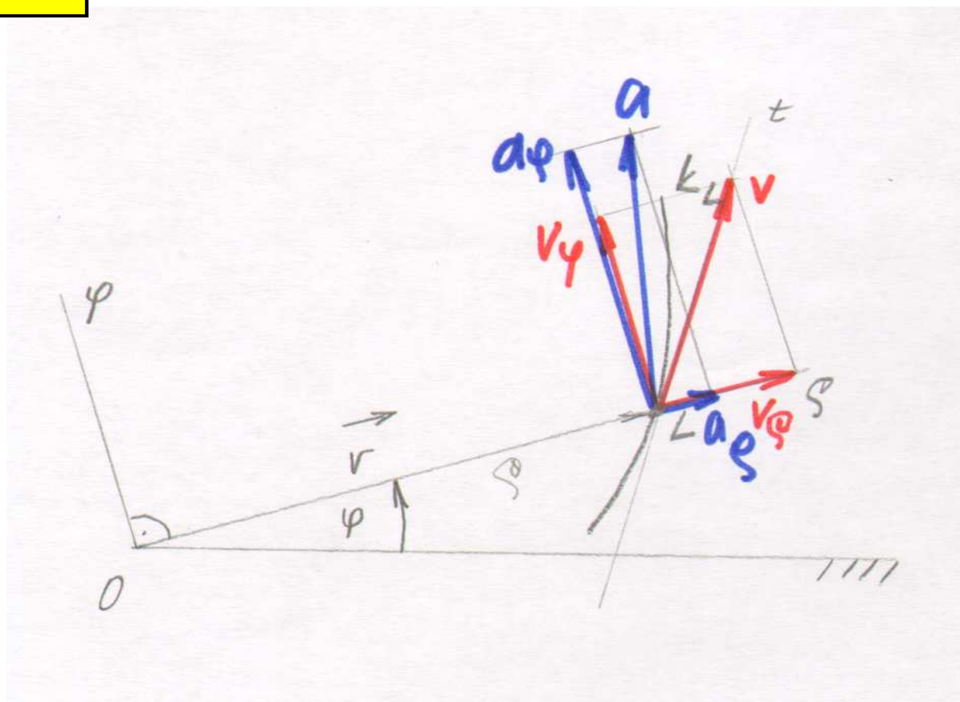
- parametric equation $\rho = \rho(t), \varphi = \varphi(t), z = z(t)$
- unit vector $\vec{e}_\rho, \vec{e}_\varphi, \vec{e}_z$
- velocity $\vec{v} = v_\rho \vec{e}_\rho + v_\varphi \vec{e}_\varphi + v_z \vec{e}_z$

$$v_\rho = \frac{\mathrm{d}\rho}{\mathrm{d}t} = \dot{\rho} \qquad v_\varphi = \rho \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \rho\dot{\varphi} \qquad v_z = \frac{\mathrm{d}z}{\mathrm{d}t} = \dot{z}$$

- acceleration $\vec{a} = a_\rho \vec{e}_\rho + a_\varphi \vec{e}_\varphi + a_z \vec{e}_z$

$$a_\rho = \ddot{\rho} - \rho\dot{\varphi}^2 \qquad a_\varphi = \rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi}$$

$$a_z = \frac{\mathrm{d}v_z}{\mathrm{d}t} = \ddot{z}$$



Harmonic motion

- Motion of a particle along straight line. Repetitive motion (vibration of machines, swing of pendulum, projecting moving uniformly along the circle)
- Differential equation $\ddot{x} + \omega^2 x = 0$ and solution $x = r \sin(\omega t + \varphi_0)$
- x ... displacement, r ... amplitude of motion, ω ... frequency of motion
- φ_0 ... initial phase
- Time of period of motion …T; T=2π/ω
- Frequency of motion (number of periods per unit of time f [s^{-1}]=1 Hz, f=1/T
- velocity $v = r\omega \cos(\omega t + \varphi_0)$
- acceleration $a = -r\omega^2 \sin(\omega t + \varphi_0)$